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Quantitative thinking 1971

ACER

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commonwealth
secondary
scholarships
examination

morning session:
thursday
22 july
1971

time allowed:
two hours

quantitative thinking

instructions to candidates

This is a test of your ability to use basic mathematical principles and ideas. If a question involves terms or principles which you have not met before, these will be explained in sufficient detail to enable you to answer the question. You may also want to make use of the symbols and formulae printed on this page.

You will obtain the best possible score if you observe the following points:

(1) Work carefully through the questions in the order in which they are given.

(2) Don't waste too much time on any one question; if necessary, go on to the next question and come back to the difficult ones later. (3) If you think you know an answer, mark it—even if you are not certain you are correct. Marks will **not** be deducted for wrong answers.

(4) Make sure that you mark the letter you have chosen in the correct line on your answer sheet.

answering

Each question has four alternative answers, represented by the letters A B C D. You must choose one answer from these alternatives. Having done so, make sure you mark your answer correctly.

If you want to change an answer, erase your first mark completely. Try to avoid having to make erasures by not answering hastily. Take care that your pencil mark does **not cross** into another row or column, and that there are no marks or smudges on your answer sheet.

Now look through this booklet, but **don't start writing** until the supervisor tells you to do so.

symbols

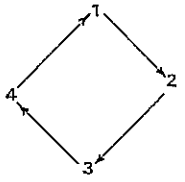
$=$ means 'is equal to'
$\neq$ means 'is not equal to'
$>$ means 'is greater than'
$\geqslant$ means 'is greater than or equal to'
$<$ means 'is less than'
$\leqslant$ means 'is less than or equal to'
$\llcorner$ indicates that the angle between the two lines is a right angle
$\angle ABC$ means 'angle ABC'
$\triangle ABC$ means 'triangle ABC'
$\overrightarrow{\ }\!\!\!\!\overrightarrow{\ }$ indicates that the two lines are parallel
$\times\!\!=$ indicates that the two lines are equal

formulae

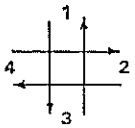
circumference of a circle
$=$ $2\pi \times$ radius, i.e. $C = 2\pi r$
area of a circle
$=$ $\pi \times$ square of radius, i.e. $A = \pi r^2$
area of a rectangle
$=$ length $\times$ breadth, i.e. $A = l \times b$
area of a triangle
$=$ $\frac{1}{2} \times$ base $\times$ height, i.e. $A = \frac{1}{2}b \times h$

Questions 1–3 refer to the following information:
The New Face Cosmetic Company has its sales territory divided into four regions, and one saleswoman works in each region. If, at the end of each month, sales are going well, all saleswomen remain where they are for another month. However, if sales are not going well, all saleswomen are moved to a different region. One of the four schemes described below is used at the end of each month.

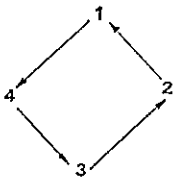
Scheme W: The saleswoman from region 1 is moved to region 2, the one from region 2 is moved to region 3, the one from region 3 is moved to region 4, and the one from region 4 is moved to region 1. We could represent this scheme by the following diagram:



Scheme X can be represented by the following diagram:



Scheme Y can be represented by the following diagram:



Scheme Z: All saleswomen remain where they are.

1 If *Scheme X* were used at the end of the first month and *Scheme Y* were used at the end of the second month, the saleswoman originally in region 2 would then be in region

A 1.
B 2.
C 3.
D 4.

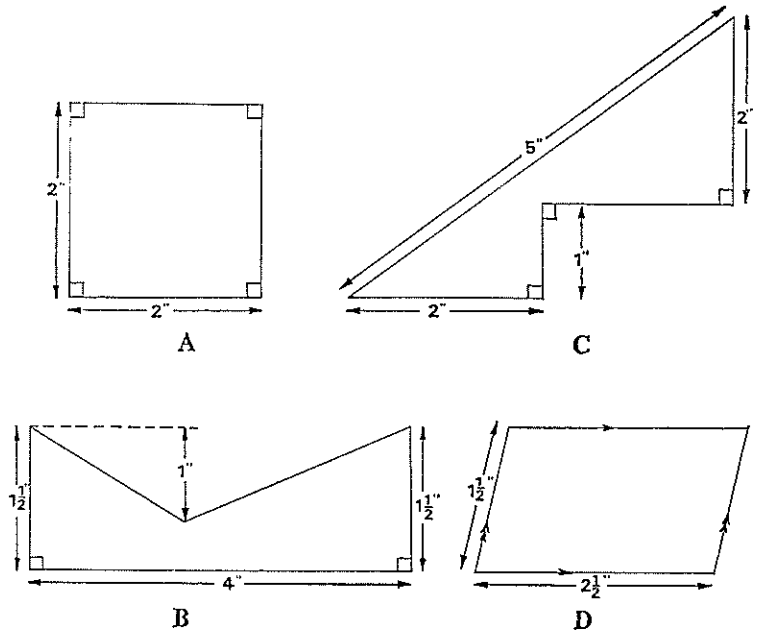
2 The final placement of the saleswomen after using *Scheme X* and then *Scheme Y* would be the same as if only

A *Scheme W* had been used.
B *Scheme X* had been used.
C *Scheme Y* had been used.
D *Scheme Z* had been used.

3 If *Scheme X* were used to shift the saleswomen from their original regions, which one of the following arrangements could then be used to return them to their original regions?

A *Scheme Y* and *Scheme X*
B *Scheme W* used twice
C *Scheme X* used twice
D *Scheme W* and *Scheme X*

4 Which one of these figures has an area different in magnitude from the others?



Questions 5–7 refer to the following information:
A model of a house and garden was built to a scale in which one inch represents one foot.

5 The floor area of the model house was 4 sq ft, so the real house has a floor area of

A 16 sq ft.
B 64 sq ft.
C 144 sq ft.
D 576 sq ft.

6 The length of fence around the house is 400 ft. The area of the model grounds

A is 400 sq in.
B is 1,000 sq in.
C is 1,000 sq ft.
D cannot be determined from the information.

7 The model has a swimming pool containing 10 gallons of water. How many gallons of water will the real swimming pool hold? (There are 6·25 gallons in one cubic foot.)

A 6,250
B 10,000
C 14,400
D 17,280

Questions 8 and 9 refer to the following information:
The following are the first 6 terms of the Fibonacci series, in which each term after the first two terms is the sum of the previous two terms:

1, 1, 2, 3, 5, 8, . . .

8 The eighth term in the series is

A 13
B 15
C 21
D 25

9 If a, b, and c are three consecutive terms of the Fibonacci series, then $ac - b^2$ is equal to one of the following. Which is it?

A 1 or 0
B 1 or − 1
C − 1 or 0
D 0 only

Questions 10–12 refer to the following information:
A set of numbers, namely: 1, 5, 9, 13, 17, . . . , consists of all the numbers of the form $4x - 3$, where x is a positive integer. The product of any two numbers **in this set** is a number contained in the set.

A ' pseudo-prime ' **in this set** may be defined as a number which has no factors **in this set,** except itself and 1. For example, the numbers 5, 9, 13 are all pseudo-primes.

10 Which one of the following is a number contained in the set?

A $9 + 17$ **C** $37 + 17$
B 5×21 **D** 3×9

11 The first number in this set which is **not** a pseudo-prime is

A 16 **C** 25
B 21 **D** 33

12 Which one of the following represents a factorization of 945 into pseudo-primes of this set?

A 5×189 **C** $3 \times 5 \times 7 \times 9$
B 21×45 **D** $5 \times 9 \times 21$

Questions 13 and 14 refer to the following table:

$$1 = 1$$
$$3 + 5 = 8$$
$$7 + 9 + 11 = 27$$

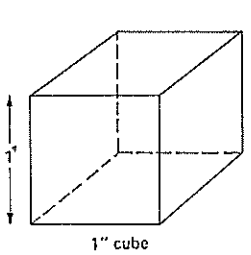
13 If we could generalize, we would expect the total of the next line of the table to be equal to

A 7^2 **C** $2^3 \times 3^2$
B 4^3 **D** 3^4

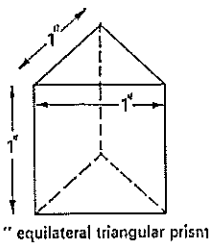
14 Which line of the table would give a total of 343?

A the 7th line
B the 8th line
C the 9th line
D No line of the above table would give a total of 343.

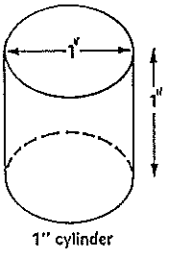
Questions 15–17 refer to the following information:
Whether or not a solid object can pass through a given hole depends not only on the size of the object but also on the way in which it is held. If, by tilting or rotating or both, an object can pass through a hole so that it touches the hole at all points on the boundary of the hole, it is said to 'just fit' the hole (when held that way). Assume that if an object just fits a hole, then it will pass through the hole when held that way.
Consider the following solid objects, together with the 1″ square hole, shown below.



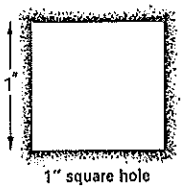
1″ cube
(i)



1″ equilateral triangular prism
(ii)



1″ cylinder
(iii)



1″ square hole

15 Which of the objects can pass through the hole if it is held in a suitable way?

A (i) only
B (i) and (ii) only
C (i) and (iii) only
D all of them

16 Which of the objects will always pass through the hole no matter how it is held?

A none of them
B (ii) and (iii) only
C (iii) only
D all of them

17 Which of the objects can be made to 'just fit' the hole if it is held in a suitable way?

A (i) only
B (i) and (ii) only
C (i) and (iii) only
D all of them

Questions 18–20 refer to the following information:

The operation $*$ is used as follows: $x * y$ means x^y

for example $x * 2 = x^2$

$3 * 2 = 3^2 = 9$

18 If $x = \frac{1}{2}$ and $y = 3$, $x * y$ equals

A $\frac{3}{2}$
B $\frac{1}{6}$
C $\frac{1}{8}$
D $\frac{1}{9}$

19 If $x * 2 = x$, then the value of x is

A -1 only
B 1 or 0
C -1 or 0
D 0 only

20 Let $w = x * y$

If $x = z$, and $y = 2$, then $w * 3$ equals

A $3z^2$
B $2z^3$
C z^6
D z^5

Questions 21–24 refer to the following information:

The 'top fifty' is displayed on a board in order of popularity. Each song has alongside it one of the numbers 1–50 to show its position. The numbers 1–50 are made up from a collection of plastic figures, each plastic figure representing one of the digits 0–9. Each number is made from one or two plastic figures, as required.

21 How many plastic figures are needed for the display?

A 50
B 89
C 90
D 91

22 Which one of the following statements about the relative frequency with which the plastic figures occur in the display is true?

A 5 occurs more often than 2
B 1 occurs more often than 3
C 0 occurs more often than 8
D 4 occurs more often than 6

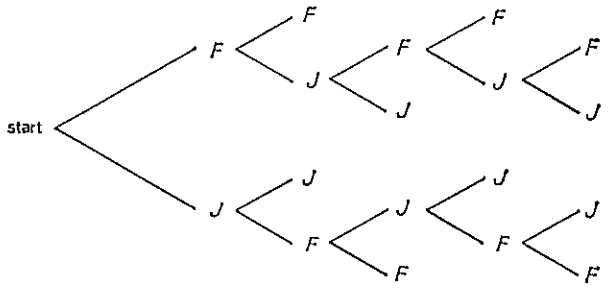
23 If the collection of plastic figures from which the display was prepared originally contained equal numbers of each of the digits 0 to 9, what is the minimum number of plastic figures in the collection?

A 50
B 90
C 150
D 180

24 If, instead of displaying them simultaneously, the songs together with their numbers were shown one at a time, how many plastic figures would be required?

A 14
B 15
C 19
D 20

Questions 25–27 refer to the following information:
Jill and Fred arrange to play a tennis tournament under the following rules:
The first person to win 2 games in a row **or** a total of 3 games wins the tournament.
The following diagram shows the various ways in which the tournament could occur.
(J represents a win of a game for Jill ; F represents a win of a game for Fred.)



25 How many ways can the tournament possibly occur?

A 4
B 10
C 18
D none of the above

26 What is the largest possible number of games that can be played in the tournament?

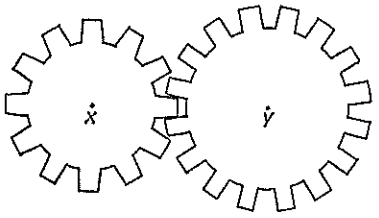
A 5
B 18
C 19
D none of the above

27 If it is known that Jill wins the tournament although she lost at least one game, in how many ways could the tournament have occurred?

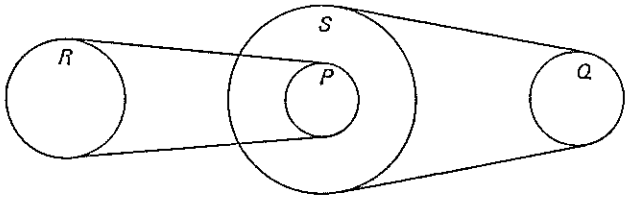
A 4
B 5
C 9
D none of the above

28 Wheel x has 12 teeth and y has 16 teeth. When x turns through 20 revolutions, how many revolutions does y turn through?

A 15
B 20
C $26\frac{2}{3}$
D none of the above



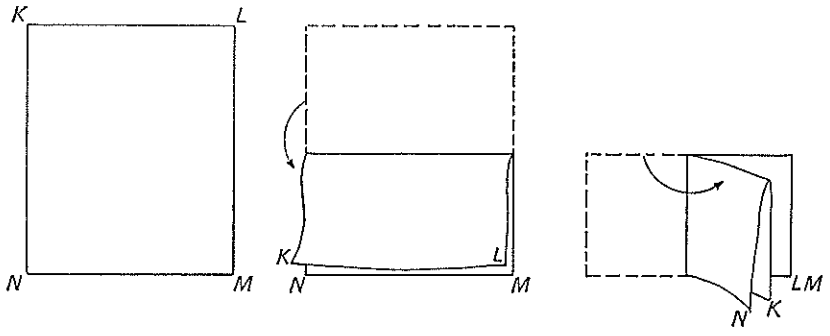
29 Wheels P, Q, R, S have circumferences of 1, 2, 3, 4 units respectively. They are connected by non-slipping belts as shown, P and S being attached to the one axle so that they rotate together.



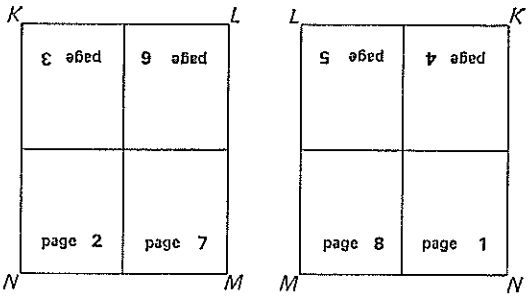
When R turns through 12 revolutions, how many revolutions does Q turn through?

A 8
B 18
C 72
D none of the above

Questions 30 and 31 refer to the following information:
An eight-page newsletter consists of a single sheet printed on both sides and folded as shown:



The arrangement of a printed sheet is as shown:



30 Which one of the following statements is correct?

A When folded, some pages will be upside-down.
B When folded, page 5 will appear before page 4.
C The pages are arranged wrongly—page 2 should be on the back of page 1, page 3 on the back of page 2, and so on.
D There is no error in the layout.

31 Suppose that a sheet is printed as shown above and folded in the same way, but before it is folded, it is turned over so that the reverse side is uppermost instead of the front uppermost. In what order will the pages appear after folding?

A 2 7 8 1 4 5 6 3
B 5 6 7 8 1 2 3 4
C 7 8 5 6 3 4 1 2
D 8 7 6 5 4 3 2 1

Questions 32–35 refer to the following information:
A dealer in a pet shop has an aquarium stocked with four kinds of fish—Angels, Guppies, Carp, and Redfin.

The dealer does not know how many fish he has in the aquarium, but he remembers that he bought twelve Angels, that he bought twice as many Guppies as Carp, and the number of Redfin he bought was two less than the number of Guppies. If he had ordered two more Carp, he would have had the same number of these as he has of Angels.

The fish can all move very fast in the water. If a customer wishes to buy a particular kind of fish, the dealer scoops the nearest fish out with a net, and if it is the wrong kind, drops it back and tries again.

32 The number of Guppies in the aquarium is

A 20 **C** 24
B 18 **D** 28

33 A customer wishes to buy a Redfin. The chances that the dealer will scoop out one of these on his first try are

A 1 in 18 **C** 1 in 4
B 3 in 10 **D** 3 in 4

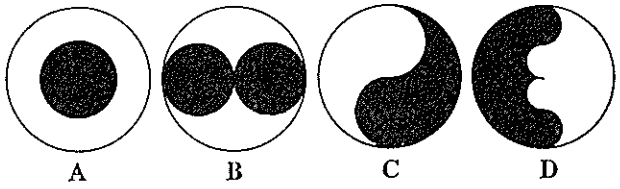
34 The customer decides he would like an Angel as well as a Redfin. If the dealer has already scooped out a Redfin, the chances that he will then scoop out an Angel on his next try are

A 12 in 60 **C** 12 in 59
B 30 in 60 **D** 3 in 59

35 Suppose that, after several attempts, the dealer is getting somewhat frustrated with trying to catch the right fish. The customer in sympathy says, 'I'll settle it by taking either an Angel or a Guppy or a Redfin, but please, no Carp!' What are his chances of getting what he wants?

A 1 in 50 **C** 10 in 60
B 50 in 396 **D** 5 in 6

36 All except one of the circles **A**, **B**, **C**, or **D** have half their area shaded. Which is the exception?



Questions 37–39 refer to the following information:
Fred has three clean bottles, one large, one medium, and one small, called *L*, *M*, *S* respectively. He also has a large tub of water.

L can hold $1\frac{1}{2}$ times as much as *M*.

M can hold $1\frac{1}{2}$ times as much as *S*.

An '*f*' in front of the name of a bottle means that it is filled from the tub. For example, *fS* means fill *S* from the tub.

Similarly an '*e*' in front of the name of a bottle means it is emptied into the tub.

A hyphen between the names of two bottles means that water is poured from the first named bottle into the second until the first one is emptied **or** the second one filled.

37 Which one of the following would leave *S* half full?

A *fM, M-S, eS, M-S*
B *fM, M-L, fS, S-L*
C It can be done, but not by either of the ways shown above.
D It can't be done with the means available.

38 Which one of the following would leave *M* half full?

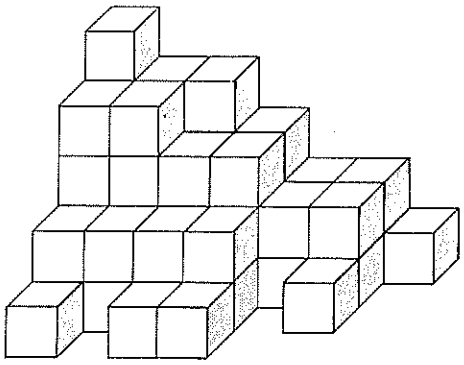
A *fL, L-S, L-M, L-S, S-M*
B *fS, S-M, fL, L-M, M-S*
C It can be done, but not by either of the ways shown above.
D It can't be done with the means available.

39 Which one of the following would leave *L* half full?

A *fL, L-M, M-S, M-L*
B *fM, M-L, fS, S-L, L-M*
C It can be done, but not by either of the ways shown above.
D It can't be done with the means available.

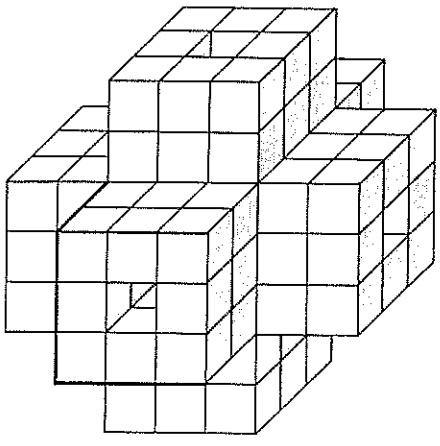
40 How many blocks are there in the stack shown? (The hidden sides of the stack are straight, and there are no 'holes'.)

A 49
B 50
C 51
D none of the above

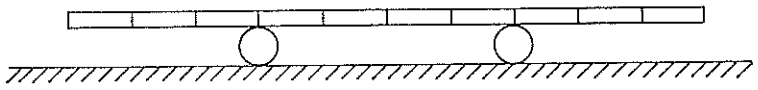


41 How many blocks are there in the model shown? (The model is symmetrical, and the 'tunnels' go right through.)

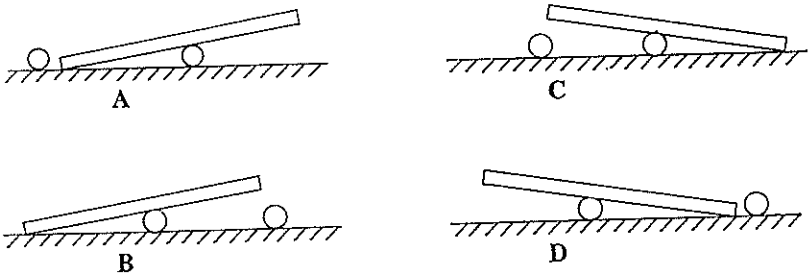
A 96
B 116
C 120
D 164



42 The diagram shows a 10″ bar resting on two rollers which rest on a table. Each roller has a circumference of 1″. (*NB* The diagrams are not to scale.)



If the bar is rolled to the right (without slipping), one of the following situations occurs. Which one?



Questions 43–45 refer to the following information:

If two numbers are combined using the operation $\square$, then the following rule is applied.

Rule : $x \square y$ means $(x + y)^2$

for example $3 \square 4 = (3 + 4)^2 = 7^2 = 49$

43 If $z = x \square y$, which one of the following statements is always **false**?

A $z < 0$
B $z > 1$
C $0 < z < 1$
D $\sqrt{z}$ is an integer

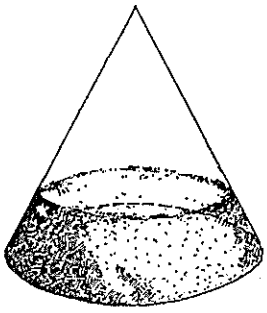
44 If $x = 2ab$ and $y = ac$, then $x \square y$ equals

A $a^2 (4b^2 + c^2)$
B $4a^2 (b^2 + c^2)$
C $a^2 (4b^2 + 4bc + c^2)$
D $4a^2 (b^2 + 2bc + c^2)$

45 If x and y are both non-negative, then $x^2 = x \square y$

A for all values of y, only if $x = 0$
B for all values of x, only if $y = 0$
C only if both x and $y = 0$
D for no values of x or y

46 The diagram shows a circular cone containing half of its own volume of a liquid. Suppose that the top surface of the liquid has area X units.
When the cone is inverted, suppose that the top surface of the liquid has area Y units.
Which one of the following statements is true?



A $X = Y$
B $X < Y$
C $X > Y$
D There is not enough information to decide between the above possibilities.

Questions 47–49 refer to the following information:
Study the following example of a statement and its converse.
Statement: If Billy is a goat, then Billy has four legs.
Converse: If Billy has four legs, then Billy is a goat.
In this example the statement is true, but its converse is false. Below are several mathematical statements. Read them and use the following key to classify them:

A The statement is true and the converse is true.
B The statement is true and the converse is false.
C The statement is false and the converse is true.
D The statement is false and the converse is false.

47 All squares are quadrilaterals with all angles right angles.

48 If $(x - 2)(x - 3) = 0$, then $x = 3$

In question 49, N, a, and b are all whole numbers.

49 If $\frac{N}{a}$ and $\frac{N}{b}$ are whole numbers, then $\frac{N}{ab}$ is a whole number.

Questions 50–52 refer to the following unlikely story:
At 10 o'clock one morning an electric clock was switched off and removed to allow a wall to be painted. An hour and a half later the clock was replaced without adjustment but the plug was wrongly inserted in the socket with the result that the hands moved at the correct speed but in the wrong direction. Some time later it was noticed that the clock showed the time as 7.30.

50 What was the correct time?

A 3.00 p.m.
B 2.00 p.m.
C 1.30 p.m.
D 12.30 p.m.

51 If the mistake had not been realized, then some time later the clock would have shown the correct time. When would this have occurred?

A 5.15 p.m.
B 4.45 p.m.
C 4.30 p.m.
D 4.00 p.m.

52 At a union meeting a vote was called on a strike motion. On a show of hands the chairman announced that the motion was carried by a majority equal to $\frac{1}{4}$ of the number opposing the motion. Then one member said the vote should have been secret, so another vote was taken. Twelve of the voters decided to change from 'strike' to 'don't strike', and none went the other way. If the motion was lost by one vote, how many voted at the meeting?

A 92
B 115
C 207
D none of the above

Question 53 refers to the following information:
The diagram shows an 'hour-glass' consisting of two glass bulbs joined together and partially filled with sand. When the glass is turned over the sand runs out at a constant rate and takes 10 minutes to pass from the upper bulb to the lower one.

The following sequence of operations is performed:

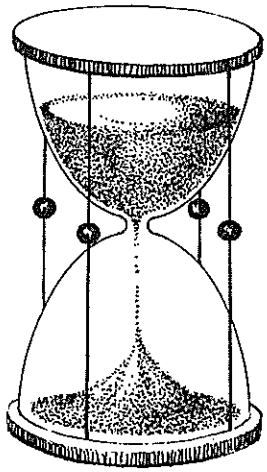
Starting with all the sand in the lower bulb, the glass is turned over.

After 7 minutes the glass is turned over again.

After a further 4 minutes the glass is turned over again.

After a further 8 minutes the glass is turned over again.

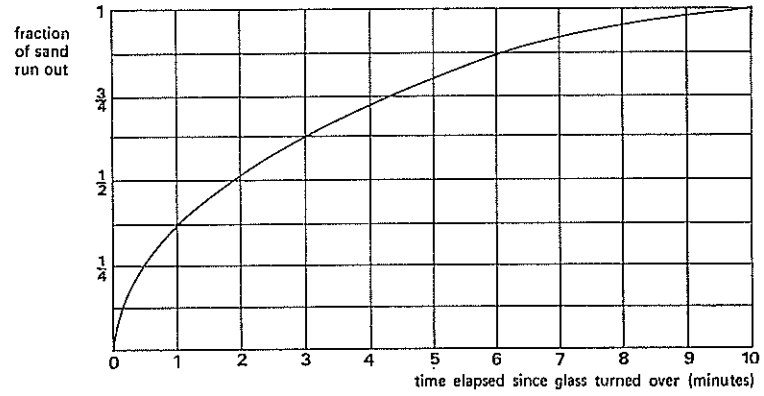
After a further 9 minutes the glass is turned over again.



53 How much time elapses after the last operation before all the sand has run out?

A 10 minutes
B 9 minutes
C 2 minutes
D 1 minute

Questions 54 and 55 refer to the following information:
Another 'hour-glass' also takes 10 minutes to empty. However, because of the way it is made, the sand runs quickly at first and then moves slowly, as shown by the graph below.



54 Starting with all the sand in the upper bulb, what fraction of the sand will have run out after 6 minutes?

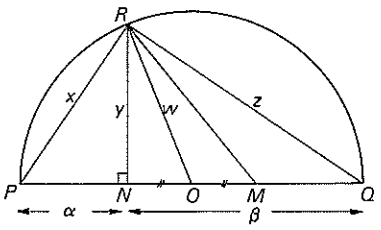
A $\frac{1}{8}$
B $\frac{3}{5}$
C $\frac{3}{4}$
D $\frac{7}{8}$

55 Starting with all the sand in the lower bulb, the glass is turned over. After 3 minutes it is turned over again. How long will it be from this time until all the sand is in the lower bulb?

A 3 minutes
B 5 minutes
C 7 minutes
D 9 minutes

Questions 56–58 refer to the following information:

PQ is a diameter of a circle whose centre is O, and R is any point on the semi-circle shown. x, y, w, z, α, and β refer to the lengths of PR, NR, OR, QR, PN, and NQ, respectively.



56 $\frac{\alpha+\beta}{2}$ is equal to one of the following. Which is it?

A w
B x
C y
D z

57 $\sqrt{\alpha\beta}$ is equal to one of the following. Which is it?

A w
B x
C y
D z

58 NM is equal to

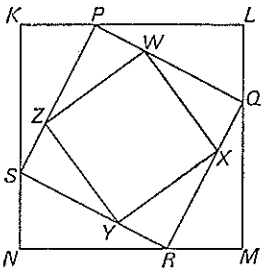
A $\frac{1}{\alpha}-\frac{1}{\beta}$
B $\frac{\beta-\alpha}{2}$
C $\beta-\alpha$
D none of the above

59 $0{\cdot}2\dot{3}$ (that is $0{\cdot}233333\ldots$) is equal to

A $\frac{21}{90}$
B $\frac{21}{99}$
C $\frac{23}{99}$
D $\frac{23}{90}$

Questions 60 and 61 refer to the following information:

In the diagram, the vertices of square $PQRS$ are points of trisection of the sides of square $KLMN$ (that is, $KP=\frac{1}{3}KL$, etc.), and the vertices of square $WXYZ$ are points of trisection of the sides of square $PQRS$. KL is 3 units long.



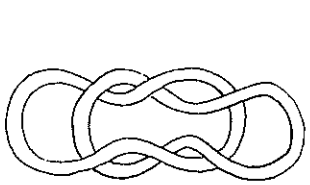
60 What is the area of square $PQRS$?

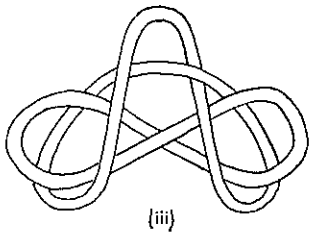
A 4 square units
B 5 square units
C 6 square units
D 7 square units

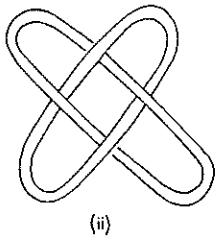
61 What is the area of square $WXYZ$?

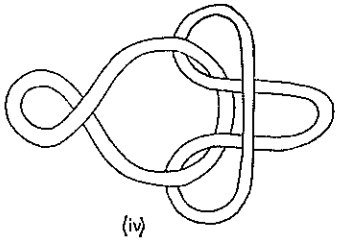
A 1 square unit
B $1\frac{7}{9}$ square units
C $2\frac{7}{9}$ square units
D 4 square units

62 Each diagram shows two loops twisted together. Some of the loops are linked so that they cannot be separated. Which pairs of loops can be separated?









A (i) and (ii) only
B (iii) and (iv) only
C (i), (iii), and (iv) only
D (iii) only

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