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COMMONWEALTH SECONDARY SCHOLARSHIPS
EXAMINATION FOR ONE-YEAR SCHOLARSHIP 1965

QUANTITATIVE THINKING

Morning Session, Thursday, 8th October, 1964

Time allowed : 2 hours

TEST BOOKLET. TO BE HANDED IN WITH YOUR ANSWER SHEET.

INSTRUCTIONS TO CANDIDATES.

This is a test of your ability to use basic mathematical principles and ideas.

If a question involves terms or principles which you might not have met before, these will be explained in sufficient detail to enable you to answer the questions concerned.

In addition, a selected list of symbols and simple formulae is printed on page 1 of this booklet.

You are strongly advised to observe the following points :

1. Work carefully through the questions in the order in which they are given.
2. Do not waste too much time on any one question ; if necessary, go on to the next question and come back to the difficult ones later.
3. If you think that you know an answer write it down even if you are not certain that it is correct.
4. Make sure that you print each answer in the correct space on the answer sheet.

ANSWERING.

In some cases you will be required to select one answer from four or five alternatives and to indicate your answer by printing the appropriate letter (A, B, C, D, &c.) on your Answer Sheet. In a few cases you will be required to select more than one of the alternatives. When this occurs clear instructions will be given in the question concerned.

In other cases you are not required to choose from alternatives but to work out the answer and write it in the correct space on your Answer Sheet. In those questions where it is necessary to specify a unit in the answer the name of the unit has been written on the Answer Sheet for you.

Example X.

One yard of string is divided into two equal pieces. How long is each piece ? There are many ways of writing the correct answer to this question. However, if you look on the Answer Sheet next to Example X you will see that the answer required is in feet and inches (i.e., 1 ft. 6 in.).

If you wish to change an answer, cross it out and print your new answer beside it.

Make any notes or calculations on this Test Booklet. Write only your answers on the Answer Sheet.

NOW look through this examination paper but do not start writing until the supervisor tells you to do so.

SYMBOLS :

- = means 'is equal to'.
 < means 'is less than'.
 > means 'is greater than'.
 \perp indicates that the angle between the two lines is a right-angle.
 $\angle ABC$ means 'angle ABC'.

FORMULAE :

- Circumference of a circle = $2\pi \times \text{radius } (2\pi r)$.
 Area of a circle = $\pi \times \text{square of radius } (\pi r^2)$.
 Area of a rectangle = length \times breadth ($l \times b$).
 Area of a triangle = $\frac{1}{2} \times \text{base} \times \text{height } (\frac{1}{2} b \times h)$.
 Volume of a cuboid = length \times breadth \times height ($l \times b \times h$).
-

1. A boxful of sweets weighs 70 oz. When it is exactly half-full it weighs 38 oz. What is the weight of the box?
2. In this question the symbols Θ , \mathcal{E} , Δ , all represent real numbers.

$$\text{If } \frac{\Theta \times \Delta}{\mathcal{E}} = \frac{\Theta \times \Theta \times \Delta \times \Delta}{Y}$$

then Y is equal to

- A. $\mathcal{E} \times \mathcal{E}$
 B. $\mathcal{E} \times \mathcal{E} \times \mathcal{E}$
 C. $\mathcal{E} \times \Delta \times \Delta$
 D. $\mathcal{E} \times \Theta \times \Theta$
 E. $\mathcal{E} \times \Theta \times \Delta$

Questions 3-5.

Consider a new kind of multiplication, restricted to the symbols 0, 1, 2, 3, and for which the multiplication table is shown below. If we take the first factor from the top row, and the second factor from the left-hand column, then the result of multiplication is found in the cell that is common to that particular row and column. Thus $3 \times 1 = 3$, because the number in the last column and the second row is 3, as underlined. Similarly $2 \times 3 = 2$ because the number in the second to last column and last row is 2, as underlined.

\times	0	1	2	3
0	0	0	0	0
1	0	1	2	<u>3</u>
2	0	2	0	2
3	0	3	<u>2</u>	1

When a and b are used in the following questions they represent the symbols 0, 1, 2, 3.

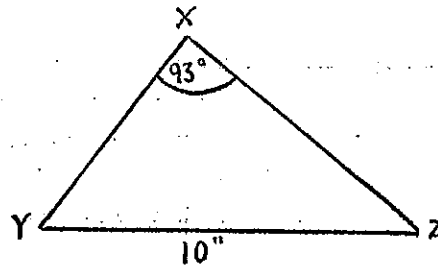
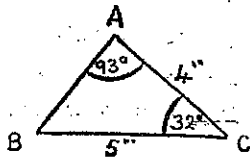
3. The relationship $a \times b = b \times a$
- A. is true only if $a = b$.
 B. is true only if $a = 0$ or $b = 0$.
 C. is true for all possible values of a and b .
 D. is not true for any value of a or b .
4. If a is not equal to 0 for what value of a will $a^2 = 0$?
5. Find the value of 3^3 using the above method of multiplying.
6. Of 30 people boarding a west-bound aircraft at Essendon airport, 20 said they would be visiting Alice Springs, 22 said they would be visiting Perth, and 23 said they would be visiting Adelaide. What is the least possible number of people planning to visit all three places?
7. In an examination four candidates score marks a, b, c, d . The average for the four is denoted by x . Later a fifth candidate sits for the examination and his mark e goes with the other four marks to make a new average y such that $y = 2x$.

Now study the method of finding the relationship between e and x .

Step A $a + b + c + d = 4x$
 Step B $a + b + c + d + e = 5y$
 Step C $a + b + c + d + e = 10x$
 Step D $4x + e = 10x$
 Step E $e = 6x$

Write S if you think that the working is correct. If you think that the working is not correct

Questions 8-10.



Two triangles have dimensions as shown. For each of the following statements write

- F. if the statement is contradicted by the information given in the diagrams ;
- T. if the statement is true for the information given in the diagrams ;
- C. if the information given in the diagram is not sufficient to determine whether the statement is true or otherwise.

8. The length of XY is less than the length of YZ .
9. $AB = 3$ inches.
10. Angle $XYZ = 55^\circ$.
11. The solid lines in Diagrams 1 and 2 show two different paths between P and Q . What is the length of the path in Diagram 2 ?

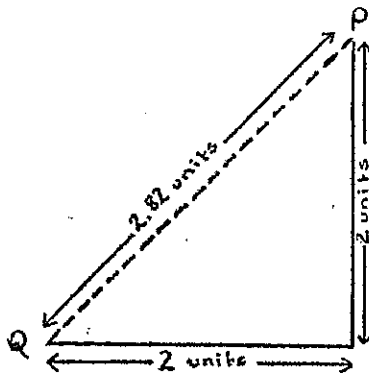


DIAGRAM 1

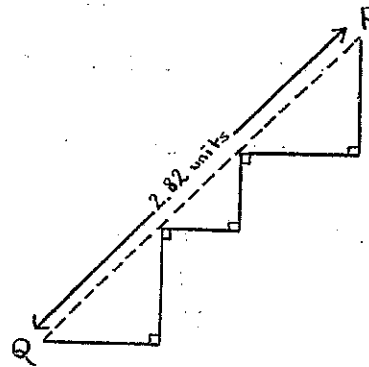
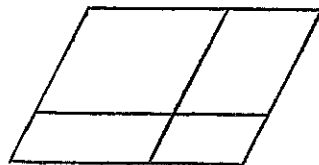


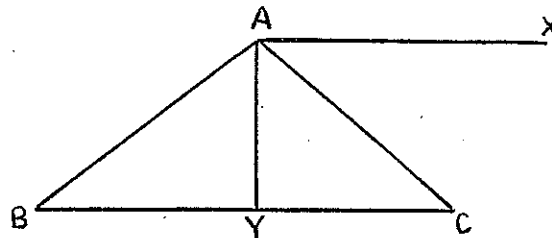
DIAGRAM 2

12.



This figure shows two sets of parallel lines. How many different parallelograms are made by these lines ?

13.



ABC is any triangle and AX is drawn parallel to BC . AY is drawn perpendicular to BC . In the following proof that $\angle YAB = \angle YAC$ which statement is the first that is not justified by the information given ?

To prove : $\angle YAB = \angle YAC$
 Proof : $\angle YBA + \angle YAB = 90^\circ$ (Statement K)
 $\angle XAC + \angle YAC + 90^\circ = 180^\circ$ (Statement L)
 $\angle YAC = \angle YAC = 90^\circ$ (Statement M)

14. This question concerns negative numbers. One of the statements given is false. Select it.
- A. The result of the addition of two negative numbers is always a negative number.
 - B. The result of the subtraction of two negative numbers is always a negative number.
 - C. The result of the multiplication of two negative numbers is never a negative number.
 - D. The result of the division of one negative number by another is never a negative number.
15. A certain fraction has a denominator which is 2 less than its numerator, x . By what fraction must this fraction be multiplied if their product is equal to one.

Questions 16-18.

In algebra the symbol $n!$ is given a specific meaning. It stands for the product

$$n \times (n - 1) \times (n - 2) \times \dots \times 3 \times 2 \times 1$$

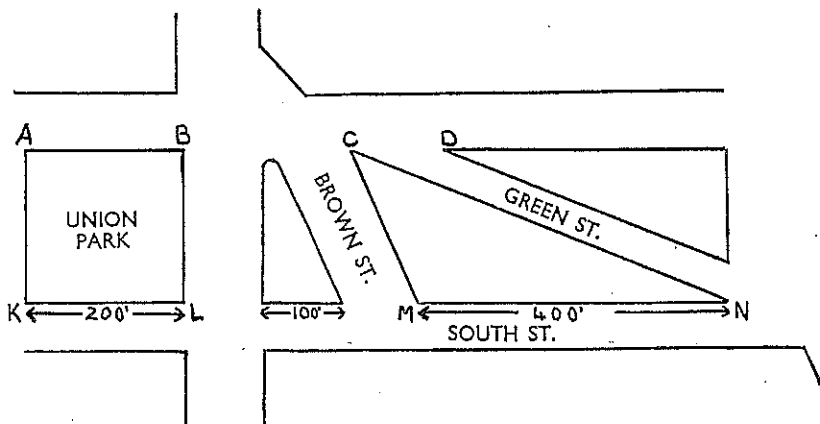
For example

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

$$7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$$

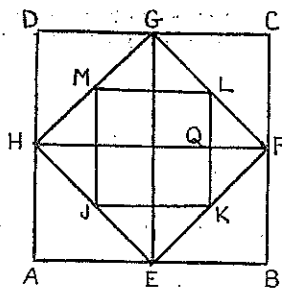
Use this information to answer the following questions :

16. Find the numerical value of $(3!)^{2!}$
17. Find the numerical value of $\frac{11!}{9!}$.
18. $\frac{(2n)!}{n!}$ is always equal to
- A. $2!$
 - B. $2n$
 - C. 2
 - D. n
 - E. $n!$
 - H. none of the above.
- 19.



In the above diagram Union Park is a rectangular garden of area 40,000 square feet. (The points A , B , C , and D are in a straight line and so are the points K , L , M , and N .) What is the area of the block bounded by Green, Brown, and South streets ?

20.



In the diagram $ABCD$ is a square and E, F, G, H, J, K, L, M are mid-points of sides as shown. What fraction of the area of $ABCD$ is the area of QKF ?

21. $a, b, c,$ and d are numbers such that c is less than b and d is greater than a . In view of this which one of the following statements is NOT true?
- $a + b$ may equal $c + d$
 - $a + d$ may equal $b + c$
 - $a + c$ may equal $b + d$
 - $a + b + c$ may equal d
 - $a + c + d$ may equal b .
22. An algebraic expression in x and y is said to be symmetrical when its value is unaltered by interchanging the x and the y . Select, from the following, the one which is NOT symmetrical:
- $x^2 + y^2$
 - $3xy$
 - $(x + y)^2$
 - $(x - y)^2$
 - $x^2 - y^2$
23. Temperatures on the fahrenheit scale (R) are obtained by multiplying the corresponding centigrade scale temperature (S) by 9, dividing this answer by 5, and then adding 32 to the result. The formula so obtained can be rearranged to give an expression for S in terms of R . Which one of the following correctly expresses this transformation?
- $S = \frac{5R - 32}{9}$
 - $S = \frac{5(R - 160)}{9}$
 - $S = \frac{5}{9}R - 160$
 - $S = \frac{5}{9}R - 32$
 - $S = 5 \frac{(R - 32)}{9}$
24. The weekly basic wage for adult males in Victoria at the 1st July, 1963, was £14 7s. The basic wage rate for adult females was stated as equal to the basic wage for adult males less 25 per cent. Which one of the following gives the annual wage of a female worker paid at the female basic wage rate?
- £ $\left[52 \left(\frac{287}{20} - \frac{25}{100} \right) \right]$
 - £ $\left(52 \times \frac{287}{20} \times \frac{100}{125} \right)$
 - £ $\left(52 \times \frac{287}{20} \times \frac{75}{100} \right)$
 - £ $\left[\left(52 \times \frac{287}{20} \right) \div \frac{100}{125} \right]$
 - £ $\left[\frac{287}{20} \left(52 - \frac{25}{100} \right) \right]$

Questions 25, 26.

A man lost in the Simpson desert walked according to the following pattern. (\square represents a missing number.)

5 miles north, 3 miles east, 2 miles south,
1 mile west, \square miles north, 5 miles west,
7 miles south, and finally, 2 miles east,

at which point he could go no further. The man was eventually located due west of his starting point.

25. What number does the \square represent?

26. How far was the man located from his starting point?

27. The side of a square sheet of cardboard is known to be an exact number of inches in length. Which is the only one of the following that could possibly represent its area?
- 441 square inches
 - 635 square inches
 - 1,000 square inches
 - 1,728 square inches
 - 8,000 square inches

Each of the questions 28 to 31 consists of a problem followed by an answer.

Write

- if the information given in the problem is not sufficient to determine whether the answer is correct;
 - if the information is just sufficient to determine the answer and shows that the answer given is reasonable;
 - if the information is just sufficient to determine the answer and shows that the answer given is obviously incorrect;
 - if the information is more than is needed to determine the answer but shows that the answer given is reasonable;
 - if the information is more than is needed to determine the answer and shows that the answer given is obviously incorrect.
28. A man bought 500 articles when they were selling at 65s. each. A fortnight later he purchased a second lot of the articles making the average cost of both transactions 66s. per article. What was the price per article of the second lot of articles purchased?
- ANSWER—£3 7s.
29. A gramophone record which rotates at the rate of $33\frac{1}{3}$ revolutions per minute takes eighteen minutes to play. If the diameter of the inside groove is 5 inches and that of the outside groove 11 inches, find the average distance between successive grooves.
- ANSWER—0.005 inches.
30. Paper of uniform width, 1 foot, and thickness 0.005 inches is rolled on to a spool of diameter 2 inches. What length of paper has been rolled on when the diameter of the roll is 1 foot?
- ANSWER—100 feet.
31. The outside area of a circular cylinder closed at one end is given by $A = \pi R(R + 2h)$.
If R and h are each made three times as great as previously how many times greater will the area become?
- 3
 - 6
 - 9
 - 18
 - 27

Questions 32–34.

Two parallel lines have no intersection; they separate a plane into 3 regions. If a third line is drawn to intersect the first two, there are 2 intersections and 6 regions. If a fourth straight line is drawn to intersect all the preceding lines (with no more than two straight lines passing through a common point), there are 5 intersections and 10 regions. Further intersecting lines are added according to the same rule.

Putting this information in table form:

Number of straight lines	2	3	4	5
Number of intersections	0	2	5	?
Number of regions	3	6	10	?

Complete the following statements:

32. With five lines the number of intersections is
33. With five lines the number of regions is

Questions 35-37.

Examine carefully the algebraic expressions below. In each case select the statement from the KEY which is most appropriate to the expression.

KEY

- A. true for all values of x
- B. false for all values of x
- C. false for all negative values of x
- D. false for all positive values of x
- E. false when x lies in the range 0 to 1 inclusive

35. $x < x + 2$

36. $x > 2x$

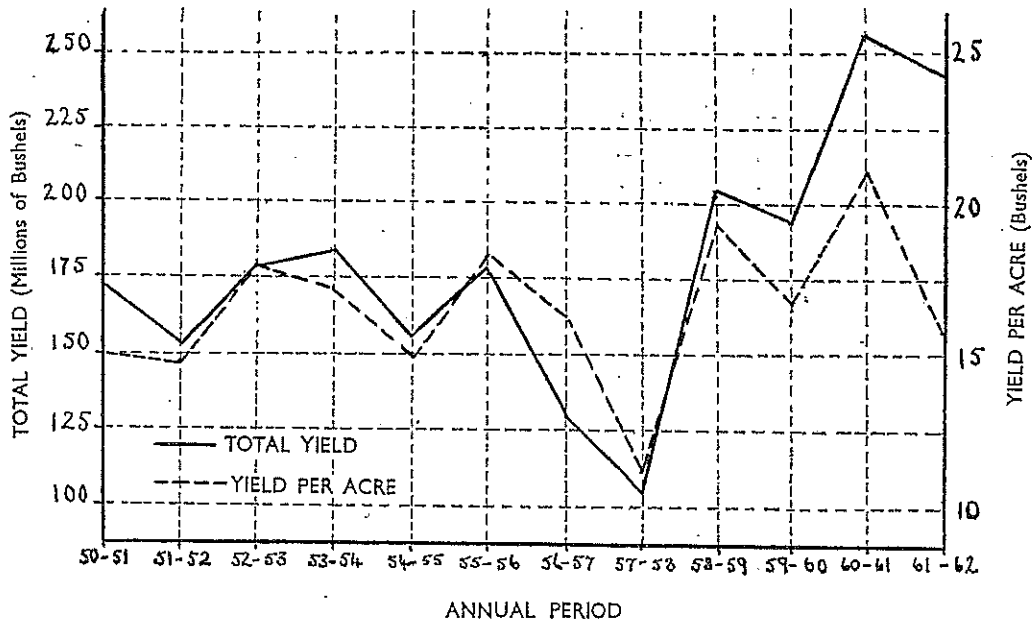
37. $x^2 > x$

38. If y is a multiple of 5 what is the next number greater than $(y + 2)$ which is exactly divisible by 5?

39. The numerators are missing in three of the following fractions. What is the result when the sum of the first two fractions is divided by the sum of the last two?

$$\frac{2x}{y} = \frac{\quad}{3y} = \frac{\quad}{y^2} = \frac{\quad}{ay}$$

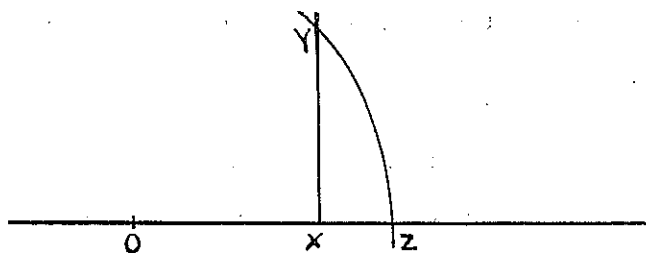
Questions 40-42 refer to the following graphs based on the production of wheat in Australia for the period 1950-51 to 1961-62.



One graph shows the total yield of wheat for each annual period. The other graph shows the average yield of wheat per acre sown for each annual period.

40. The average number of acres of wheat sown per year in Australia is approximately
- A. 100,000 acres.
 - B. 500,000 acres.
 - C. 1,000,000 acres.
 - D. 10,000,000 acres.
 - E. 150,000,000 acres.
41. For which annual period was the number of acres of wheat sown the least?
42. For which annual period was the number of acres of wheat sown the greatest?

49.



In mathematics we often represent numbers by points on a line in such a way that the distances between points representing consecutive whole numbers are equal. In the above diagram

- O represents the number zero
- X represents the number 1
- $OX = OY$
- angle $OXY = 90^\circ$
- YZ is the arc of a circle whose centre is O.

Which one of the following numbers does Z represent ?

- A. 1.5
 - B. $\sqrt{2}$
 - C. $1\frac{1}{2}$
 - D. $\frac{\pi}{2}$
 - E. none of the above
50. If n is a positive whole number greater than one, select the statement concerning the number $(n - 1) \times n \times (n + 1)$ that is always true.
- ($n - 1) \times n \times (n + 1)$ is always
 - A. exactly divisible by $2n$.
 - B. greater than 6.
 - C. exactly divisible by 3.
 - D. greater than n^2 .
 - E. not less than n^4 .

51.

x	$\frac{1}{2}$	1	2	3
y	$\frac{3}{4}$	3	12	27

} That is when $x = \frac{1}{2}, y = \frac{3}{4}$; &c.

The above values of x and y are related by one of the following types of formula (k is a constant) :

- Type I $y = kx$
- Type II $xy = k$
- Type III $y = kx^2$

Select the appropriate formula, and having determined which it is, find the value of k for this particular set of values of x and y .

52. Consider the problem—

“Mary is twice as old as her sister Joan is now. How old will Joan be when Mary is twenty ?”

If you consider that there is a single answer to the problem in its present form write Y.

If you consider that more information is required, study the statements below and select every one which would, by itself, provide sufficient additional information for a solution to be obtained.

- A. Mary's present age.
- B. Joan's present age.
- C. The sum of Mary's and Joan's present ages.
- D. The difference between Mary's and Joan's present ages.
- E. The time in years before Mary will be twenty.

Questions 53-58 relate to the following information :

A certain country decided to adopt a new calendar from the first day of January, 1965.

For the new calendar

One year = nine months.

One month = eight weeks.

One week = five days.

Appropriate adjustments were to be made periodically so that the calendar retained the mean solar year as its basis.

The following list shows the names adopted and the order in which they occur :

<u>Months</u>	<u>Weeks</u>	<u>Days</u>
January	unk	Wednesday
February	deuk	Thursday
March	troik	Friday
April	quark	Saturday
May	cink	Sunday
June	sik	
July	sepk	
August	ock	
September		

The first day of January, 1965, was to be recorded either by

METHOD A .. Wednesday/unk/January/1965.

or,

METHOD B .. 1/1/1/1965.

To illustrate further, Tuesday, 2nd February, on the traditional calendar would be written :

METHOD A .. Friday/sepk/January/1965.

METHOD B .. 3/7/17/1965.

53. How many days are there in seven months two weeks and one day of the new calendar ?
54. The 50th day in the year is shown on the traditional calendar as 19th February. How would the 50th day of the year be written on the new calendar using METHOD B ?
55. How would 17th June, 1965, from the traditional calendar be written using METHOD B ?
56. What date on the traditional calendar would correspond to Sunday/troik/April/1965 ?
57. Which of the following adjustments to the new calendar would by itself most successfully account for the difference between the lengths of the new calendar year and the mean solar year ?
- Add a full new month (Rolember) after September every 7th year ;
 - Add an extra day (teleday) to each month of the year ;
 - Add an extra week (nonk) to September every year ;
 - Add two extra weeks (nonk and deuk) to September every odd year and one extra week (nonk) to September every even year ;
 - Add an extra week (nonk) to September every 20th year.
58. Which two of the above adjustments would together make the average length of the new calendar year the same as that of the traditional calendar year ?
59. Oranges, at M for y shillings are dearer than apples which cost N shillings a dozen. Express this fact by writing the left-hand side to correctly complete the following :
- $>NM$
60. P , who travels 1 mile in m minutes, travels more slowly than Q whose speed is n miles per hour. Express this fact by writing the left-hand side to correctly complete the following :
- $<n$.

Questions 61-64 relate to the following information :

Relationships of the type $y = mx + c$ may be represented graphically by using parallel lines, marked with equal scales as the x and y axes. The following example illustrates how graphs of this nature are constructed.

Take the relationship $y = 2x + 1$:

Three pairs of corresponding values of x and y would be $(-2, -3)$, $(0, 1)$, $(2, 5)$, i.e., when $x = -2$, $y = -3$, &c.

When straight lines are drawn through the various pairs of corresponding points on the two axes it is found that all such lines pass through the point P . Hence, we say that the point P is the graphical representation of the relationship $y = 2x + 1$.

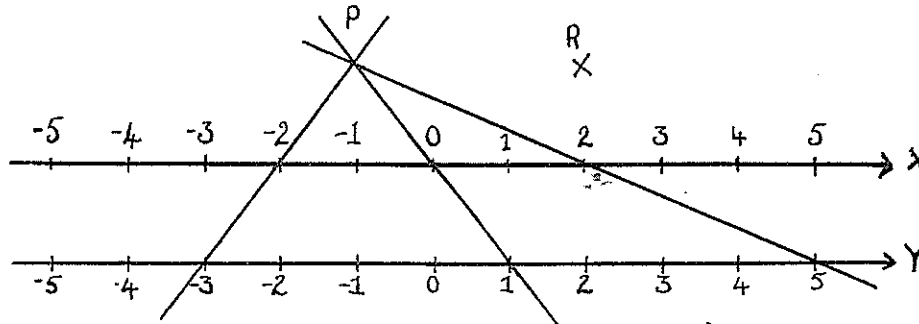


DIAGRAM I.

61. In the graph below select the point (A , B , C , D , E or H) which is the graphical representation of the relationship $y = 3x - 1$.

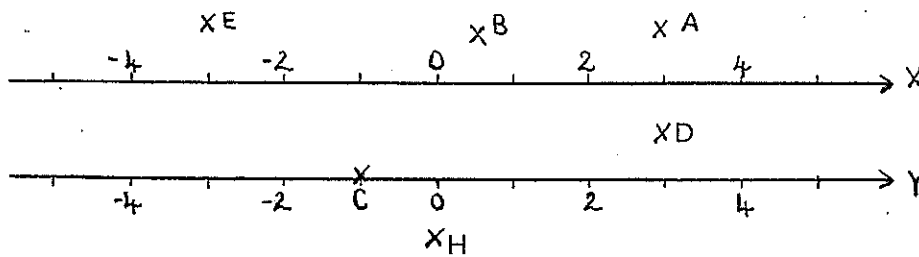


DIAGRAM II.

62. Suppose the graph of $y = 3x - 4$ is the point Q . At which point will the line perpendicular to the X axis from Q , intersect the X axis?

In diagram I, R is the graph of $y = ax + b$ where a and b are constants.

63. Use the graph to find the numerical value of y which corresponds to the value 0 of x .
64. Use the graph to find the numerical value of x which corresponds to the value 0 of y .