7-31-1969

Quantitative Thinking 1969

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COMMONWEALTH SECONDARY SCHOLARSHIPS
EXAMINATION FOR TWO-YEAR SCHOLARSHIP 1970–71

QUANTITATIVE THINKING

Morning session: Thursday 31 July 1969
Time allowed: two hours

TEST BOOKLET TO BE HANDED IN WITH YOUR ANSWER SHEET

INSTRUCTIONS TO CANDIDATES

This is a test of your ability to use basic mathematical principles and ideas. If a question involves terms or principles which you might not have met before, these will be explained in sufficient detail to enable you to answer the question concerned. In addition, a selected list of symbols and simple formulae is printed at the foot of this page.

You are strongly advised to observe the following points:

1. Work carefully through the questions in the order in which they are given.
2. Do not waste too much time on any one question; if necessary, go on to the next question and come back to the difficult ones later.
3. If you think you know an answer, mark it even if you are not certain that it is correct. Marks will not be deducted for wrong answers.
4. Make sure that you mark the letter you have chosen in the correct line on your answer sheet.

ANSWERING

For each question you will be given four alternative answers. These alternative choices will be represented by the letters A B C D. You are required to select an answer from these alternatives. Indicate your answer by putting a black pencil mark between the dotted lines across the letter representing your choice.

If you wish to change your answer you must erase your first mark completely. Try to avoid the necessity for making erasures by not answering hastily. Take care that your pencil mark does not cross into another row or column, that is it does not go outside one dotted space, and that there are no marks or smudges on your answer sheet. For example, if you choose D you should mark your answer sheet as follows:

A:  B:  C:  D:

Now look through this examination paper but do not start writing until the supervisor tells you to do so.

SYMBOLS:

= means 'is equal to', and ≠ means 'is not equal to'
> means 'is greater than' ≥ means 'is greater than or equal to'
< means 'is less than' ≤ means 'is less than or equal to'
\( \angle \) indicates that the angle between the two lines is a right angle
\( \perp \) indicates that the two lines are parallel
\( \equiv \) indicates that the two lines are equal

FORMULAE:

Circumference of a circle = \( 2\pi \times \text{radius} \), i.e. \( C = 2\pi r \)
Area of a circle = \( \pi \times \text{square of radius} \), i.e. \( A = \pi r^2 \)
Area of rectangle = \( \text{length} \times \text{breadth} \), i.e. \( A = l \times b \)
Area of triangle = \( \frac{1}{2} \times \text{base} \times \text{height} \), i.e. \( A = \frac{1}{2}b \times h \)
1 Two points $A$ and $B$ specify a line, as shown below. We shall define left and right by saying that $B$ is to the left of $A$. $X$ and $Y$ are two bugs that crawl towards the line, and are aware of the normal meanings of left and right. For which bug(s) would our definition of left and right agree with theirs?

A bug $X$ only  
B bug $Y$ only  
C both bug $X$ and bug $Y$  
D neither bug $X$ nor bug $Y$

2 The volume of these four rectangular solids is the same. Which has the smallest surface area?

A  
B  
C  
D

3 If $x$ and $y$ are whole numbers and $2x - 5y = 1$, then we can be certain that

A $y$ is an even number.  
B $y$ is an odd number.  
C $x$ is an even number.  
D $x$ is an odd number.

Questions 4 to 6 refer to the following graph and information:

The crude birth-rate is defined as the number of children born per thousand of the population, and may be obtained from the formula:

$$B = \frac{\text{number of births}}{\text{total population}} \times 1000$$

The crude death-rate is similarly expressed as:

$$D = \frac{\text{number of deaths}}{\text{total population}} \times 1000$$

The natural increase in the population is defined as:

$$I = B - D$$
4 In which of the following years was the crude birth rate the greatest?
A 1951  
B 1952  
C 1954  
D 1955

5 In which of the following years was the natural increase in population the greatest?
A 1954  
B 1959  
C 1960  
D 1961

6 In the period 1921 to 1950, the years 1931-35 had both the lowest death rate and the lowest birth rate. Of the following, which is the most plausible explanation of this low death rate?
A There were fewer old people dying.
B The total population was decreased.
C The economic depression (of 1931-35) caused more sickness and death among middle-aged people.
D Fewer babies were born and as a result fewer babies died.
Questions 7 to 11 refer to the following information:

**SYMMETRY**
A plane figure is symmetrical about a vertical axis if, when folded about that axis, the two parts of the figure become coincident, i.e. one will exactly cover the other. The square $ABCD$ is symmetrical about four axes, $PR$, $BD$, $QS$, $AC$.

![Symmetry Diagram]

**ROTATION ABOUT A LINE**
When the letter $P$ is rotated 180° about a vertical axis it looks like this:

![Rotation Diagram]

**ROTATION ABOUT A POINT**
When the letter $P$ is rotated 180° about a point $O$ it looks like this:

![Rotation Diagram]

Questions 7 to 11 refer to the letters as printed below:

A B C D E F G H I J K L M
N O P Q R S T U V W X Y Z
7 How many letters are symmetrical about a vertical line?
   A  8
   B  10
   C  11
   D  12

8 How many letters are symmetrical about a horizontal line?
   A  8
   B  9
   C  10
   D  12

9 How many letters appear the same after rotation of 180° about a point in the plane?
   A  4
   B  5
   C  6
   D  7

10 How many pairs of letters are there in which one letter, when rotated 180° about a vertical axis, is identical with the other letter?
   A  0
   B  1
   C  2
   D  3

11 How many pairs of letters are there in which one letter, when rotated 180° about a horizontal axis, is identical with the other letter?
   A  0
   B  1
   C  2
   D  4

Questions 12 and 13 refer to the following information:
The following table gives the numbers of faces \( (F) \), vertices \( (V) \) and edges \( (E) \) of the five regular polyhedra.

<table>
<thead>
<tr>
<th>Faces ( (F) )</th>
<th>Vertices ( (V) )</th>
<th>Edges ( (E) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>20</td>
<td>12</td>
<td>30</td>
</tr>
</tbody>
</table>

12 Which of the following is not a true statement?
   A  \( E \geq V \)
   B  \( E \geq F \)
   C  \( V \geq F \)
   D  \( E \leq F + V \)

13 Which of the following is the most precise description of the relationship between the numbers of faces, vertices and edges of the five regular polyhedra?
   A  \( F = 2 + E - V \)
   B  \( E \leq F + V \)
   C  \( 2 \leq \frac{FV}{E} \leq 8 \)
   D  \( \frac{3FV}{E} = 4n \), where \( n \) is a positive integer
Questions 14 to 17 refer to the following information and diagrams:
Each of the containers shown is placed in turn under a tap from which water is flowing at a steady rate. In the case of Questions 14 to 16 you are to select, from the key, the graph which best shows how the water level in that container changes with time if each container is empty at the start.

**KEY**

![Graphs A, B, C, D](image)

14

15

16

17 Now consider the regular hollow containers (i), (ii), (iii), of equal area of base.

![Containers (i), (ii), (iii)](image)

Which of the containers corresponds to the water level-time graph below?

- A one only of (i), (ii), (iii)
- B two of the containers
- C the three containers (i), (ii), (iii)
- D none of the containers (i), (ii), (iii)
Questions 18 to 22 refer to the following information:
The following table shows the number of gold, silver and bronze medals won by each of thirteen countries at the 1968 Olympic Games. In reporting the results of the Games, newspapers ranked the countries on the basis of the number of gold medals won. Where two or more countries won the same number of gold medals, the number of silver medals won was used as the basis for rating. The table below shows the first thirteen place-winners in this unofficial contest.

<table>
<thead>
<tr>
<th></th>
<th>Gold</th>
<th>Silver</th>
<th>Bronze</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. USA</td>
<td>45</td>
<td>28</td>
<td>34</td>
<td>107</td>
</tr>
<tr>
<td>2. USSR</td>
<td>29</td>
<td>32</td>
<td>30</td>
<td>91</td>
</tr>
<tr>
<td>3. Japan</td>
<td>11</td>
<td>7</td>
<td>7</td>
<td>25</td>
</tr>
<tr>
<td>4. Hungary</td>
<td>10</td>
<td>10</td>
<td>12</td>
<td>32</td>
</tr>
<tr>
<td>5. E. Germany</td>
<td>9</td>
<td>9</td>
<td>7</td>
<td>25</td>
</tr>
<tr>
<td>6. France</td>
<td>7</td>
<td>5</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>7. Czechoslovakia</td>
<td>7</td>
<td>2</td>
<td>4</td>
<td>13</td>
</tr>
<tr>
<td>8. W. Germany</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>9. Australia</td>
<td>5</td>
<td>7</td>
<td>5</td>
<td>17</td>
</tr>
<tr>
<td>10. Britain</td>
<td>5</td>
<td>5</td>
<td>3</td>
<td>13</td>
</tr>
<tr>
<td>11. Poland</td>
<td>5</td>
<td>2</td>
<td>11</td>
<td>18</td>
</tr>
<tr>
<td>12. Rumania</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>13. Italy</td>
<td>3</td>
<td>4</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>

If the countries were ranked on the basis of the total number of medals won, they would not appear in the same order.

18 Which of the following countries would rise most in the rankings if places were given on the basis of the total number of medals won?
A Hungary
B Australia
C Britain
D Poland

19 Which of the following countries would have the same rank on the basis of total medals won as it has in the table?
A Japan
B Australia
C Rumania
D none of the above

Questions 20 and 21 refer to the following additional information:
Some newspapers reported unofficial results of the Olympics on the basis of a points system, i.e. x points for a gold medal, y points for a silver medal, and z points for a bronze medal. In each case x > y > z.

20 Which of the following sets of values of x, y and z results in Japan being placed ahead of Hungary?
A 10, 6, 2
B 12, 8, 4
C 15, 5, 1
D 20, 4, 1

21 In which of the following pairs is the order incapable of being reversed, whatever the values assigned to x, y and z?
A France-Australia
B Hungary-Japan
C France-Czechoslovakia
D Britain-Rumania

22 If Italy finished with a total score of 32 under a system of scoring whereby bronze medals are worth one point each, how many points are awarded for gold and silver medals respectively?
A 4, 3
B 5, 2
C It is possible to find a simple answer but it is not A or B above.
D It is not possible to find a simple answer.
Question 23 refers to the following information:
A circular piece of paper, centre O, is cut along the radii OA and OB as indicated in the diagram. The two sectors (shaded) are then folded so that in each case the edges (OA, OB) coincide, producing a larger and a smaller cone.

23 What is the value of the ratio
\[
\frac{\text{area of curved surface of larger cone}}{\text{area of curved surface of smaller cone}}
\]
A \( \frac{1}{4} \)
B \( \frac{4}{3} \)
C \( \frac{4}{3} \)
D none of the above

Questions 24 and 25 refer to the following information:
Each of the phrases below describes two points on the surface of an object. In each question select as your answer the shape of the outline of the object that would be seen if the object was held in front of you, at a considerable distance, with the two points and your eye in the same straight line. That is, what is the (two-dimensional) outline?

24 The ends of a diameter on the base of a cone.
A triangle
B ellipse
C circle
D parabola

25 One vertex of a cube and the vertex most distant from it.
A square
B rectangle
C hexagon
D triangle

Questions 26 to 30 refer to the following information:
The set of whole numbers can be written as follows:
0, 1, 2, 3, \ldots, 9, 10, 11, \ldots, 99, 10^2, 101, \ldots, 10^5, 1001, \ldots, 10^{10}, \ldots, 10^{10^3}, \ldots
The dots (\ldots) after a number indicate that some numbers have been omitted.

26 What is the whole number next after \(10^9\)?
A \(10^{10}\)
B \(10,000,000,001\)
C \(100,000,000,001\)
D \(1,000,000,001\)

27 What is the whole number nearest to, or exactly equal to, \( \frac{10^{14}}{5000} \)?
A \(2 \times 10^{12}\)
B \(200,000,000,000\)
C \(5 \times 10^{11}\)
D \(5,000,000,000,000\)
28 What is the whole number nearest to, or exactly equal to, \( \frac{3 \times 10^6}{2} \) ?

A \( 3 \times 10^5 \)  
B \( 15^6 \)  
C \( 3 \times 10^5 \)  
D \( 1.5 \times 10^6 \)

29 What is the whole number nearest to, or exactly equal to, \( \frac{2 \times 10^4}{3} \) ?

A \( 2 \times 10^4 \)  
B \( 666,666 \)  
C \( 2000 \)  
D \( 666,667 \)

30 What is the whole number nearest to, or exactly equal to, \( 10^{100} \div 10 \)?

A \( 10^{10} \)  
B \( 10^9 \)  
C a number beginning and ending in the digit 1 but with a hundred noughts between 
D a number beginning and ending in the digit 1 but with ninety-nine noughts between

Questions 31 and 32 refer to the following information and bar graphs:
The bar graphs show the percentage of newborn females who would survive to age 20, and to age 50, according to mortality rates around AD 1550 in Europe, and in selected countries around 1950.

![Bar graphs showing survival percentages](image)

31 Consider the year 1500 in Europe. Of ten thousand females born, approximately how many of these would be expected to survive to the age of fifty or above?

A 800  
B 1250  
C 3000  
D 8000

32 Consider the total number of females that survive to age 50 in India and in Guatemala respectively. How many more females survive to age 50 in India than in Guatemala?

A There are about 5 times as many.  
B There are about 1 to 2 times as many.  
C There are not more, but less.  
D There is insufficient information to decide.
Questions 33 and 34 are based on the following information:
Various curves may be drawn between two points on a surface. The curve which has the shortest length is the geodesic.

33  State which one of the following is not true.
A  For any two points in a plane the geodesic is a straight line segment.
B  For any two points on a sphere the geodesic is an arc of a circle.
C  For any two points on a curved surface the geodesic cannot be a straight line segment.
D  For any two points on a cube the geodesic consists of one or more straight line segments.

34  A piece of paper is wrapped around the curved surface of a cylinder and a geodesic drawn on it. The paper is now unwrapped and laid flat on a plane. State which of the following is true of the line that appears on the flattened paper.
A  It is the arc of a circle.
B  It consists of two circular arcs joined in the middle.
C  It is a straight line segment.
D  Either A, B or C is correct depending on the location of the two points.

Questions 35 and 36 refer to the following graph and information:

![Figure 1]

Figure 1 is the graph of $y = ax^3$ where $a$ is a positive integer.

35  Which of the following is most likely to be the equation that Figure 2 depicts?
A  $y = -ax^3$
B  $x = ay^3$
C  $y = ax^3$
D  $x = -ay^2$
36 Which of the following is most likely to be the equation that Figure 3 depicts?
   A $y = -ax^2$
   B $x = ay^2$
   C $x = -ay^2$
   D $y = ax^2$

Questions 37 to 41 refer to the following information:
An irreducible fraction is a fraction expressed in its lowest terms; e.g. $\frac{4}{8}$ is not an irreducible fraction, but $\frac{1}{2}$ is. A proper fraction is a fraction with a value less than 1.
The Farey Sequence $F_n$ of order $n$ is defined as the ordered set consisting of $\frac{1}{n}$, the irreducible proper fractions with denominators from 2 to $n$, arranged in order of increasing magnitude, and $\frac{1}{1}$.
So $F_1$ consists of $\frac{1}{2}$ and $\frac{1}{1}$
   $F_2$ consists of $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{1}$
   $F_3$ consists of $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{2}, \frac{1}{3}$, and $\frac{1}{1}$
   $F_4$ consists of $\frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{3}, \frac{1}{4}, \frac{1}{2}$, and $\frac{1}{1}$

37 Which of the following sequences is $F_5$?
   A $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$
   B $\frac{1}{5}, \frac{1}{10}, \frac{1}{11}, \frac{1}{12}, \frac{1}{13}$
   C $\frac{1}{5}, \frac{1}{7}, \frac{1}{10}, \frac{1}{12}, \frac{1}{14}$
   D none of the above

38 For the Farey Sequence $F_n$, the second highest (and second last) term in the series is
   A $\frac{5}{7}$
   B $\frac{3}{4}$
   C $\frac{5}{8}$
   D none of the above

39 If $\frac{a}{b}$ is one term of any Farey Sequence, and $\frac{c}{d}$ is the next term, which of the following statements appears to hold?
   A $bc + ad = 1$
   B $bc - ad = 1$
   C $ad - bc = 1$
   D $ab - cd = 1$

40 Which one of the following statements about the sequence $F_n$ and the sequence $F_{n-1}$ is true?
   A All terms in $F_n$ are terms in $F_{n-1}$.
   B All terms in $F_{n-1}$ are terms in $F_n$.
   C $F_{n-1}$ has one more term than $F_n$.
   D None of the above is true.

41 It is likely that the number of terms in the Farey Sequence $F_n$, where $n > 2$, will be
   A odd always.
   B even always.
   C even if $n$ is even, odd if $n$ is odd.
   D even if $n$ is odd, odd if $n$ is even.
42. A pyramid stands on a square base of side $b$ and its vertical height is $h$. The volume is given by $\frac{1}{3} b^2 h$. If the side of the square is doubled and the height remains the same, then the volume is increased by a factor of

- A 2
- B $2\sqrt{2}$
- C 4
- D between $2\sqrt{2}$ and 4

43. In the diagram, the angle marked $a$ is the angle of slope of a face. The angle marked $b$ is the angle of slope of an edge. Which of the following statements is true regarding the magnitudes of the angles marked $a$ and $b$ respectively?

- A magnitude of $a = $ magnitude of $b$
- B magnitude of $a > $ magnitude of $b$
- C magnitude of $a < $ magnitude of $b$
- D Either A, B, or C is correct according to whether the slope of a face is equal to, less than or greater than 45°.

Questions 44 to 46 refer to the following information:
The numbers we usually use are expressed in base ten; i.e. the number $abcd$ really means $a(\text{ten})^3 + b(\text{ten})^2 + c(\text{ten}) + d(\text{one})$.
Note: one = (ten)$^0$. We can, however, express numbers in other bases. So the base five number $xyz$ means $x(\text{five})^2 + y(\text{five}) + z(\text{one})$.

44. In a certain base, the following are consecutive whole numbers:
103, 104, 110, 111, 112, 113, 114, $X$, $Y$, ...
The number $Y$ can be written

- A 115.  
- B 120.  
- C 121.  
- D 122.
45 \( p, q, r \) represent three different digits, so \( pqr \) represents a three-digit number. If \( pqr \) is a perfect square in base \( s \), then \( s \) must be
A two.
B three.  
C more than three.
D none of these; it is negative.

46 We usually define the number \( xyz \) in base \( r \) to be equal to \( xr^2 + yr + z \). For this question, however, we shall define the number \( xyz \) in base \( r \) to be equal to \( x + yr + zr^2 \). Using this notation, the number representing three dozen could, in the base of some whole number, be written
A 012.
B 021.  
C 122.
D 311.

Questions 47 to 51 refer to the following information:
In the following questions a fraction is defined as any number that can be named by the form
some natural number
some natural number

the natural numbers being 1, 2, 3 etc. Thus \( \frac{1}{2}, \frac{1}{3}, \frac{5}{2}, \frac{4}{3} \) are examples of fractions. Note that \( \frac{2}{3} \) and \( \frac{3}{2} \) also equal natural numbers. We assume that sums, products and differences (larger minus smaller) of fractions are themselves fractions. Also we assume that sums, products and differences of natural numbers are natural numbers. We now define a 'true fraction' as a fraction that is not equal to any natural number. Thus \( \frac{2}{3} \) and \( \frac{3}{2} \) are true fractions, while \( \frac{2}{3} \) and \( \frac{3}{2} \) are not.

Five propositions and respective proofs are now presented dealing with fractions. Steps in each proof are labelled A, B and C. For each proof you are to give the letter corresponding to the step containing the first error in reasoning made in the proof. If no error is made (and the proof is correct), you should write D on your answer sheet.

47 **Proposition**: The sum of a true fraction and a natural number is a true fraction.

**Proof**: If the sum of the true fraction \( f \) and the natural number \( n \) is the natural number \( m \), then \( f + n = m \)  
And \( m - n = f \)

But this last statement contradicts the assumption that the difference between two natural numbers is a natural number  
Therefore \( m \) cannot be a natural number, and the sum of a true fraction and a natural number must be a true fraction  
This is a correct proof

48 **Proposition**: The sum of two true fractions is a true fraction.

**Proof**: \( \frac{1}{2} \) is a true fraction and \( \frac{1}{3} \) is a true fraction
The sum of \( \frac{1}{2} \) and \( \frac{1}{3} \) is \( \frac{5}{6} \) which is a true fraction  
So the sum of these two true fractions is a true fraction  
Similarly the sum of two true fractions is a true fraction  
This is a correct proof
49 Proposition: The product of a true fraction and a natural number is a true fraction.

Proof: The true fraction \( \frac{p}{q} \) may be written \( \frac{np}{q} \) where \( p \) and \( q \) are both natural numbers, and \( q \) is not a factor of \( p \).

If \( n \) is a natural number, then the product of a true fraction and a natural number may be written \( \frac{np}{q} \).

\( np \) is the product of two natural numbers and so is a natural number which may be written \( m \).

So the product of a true fraction and a natural number \( n \) is a true fraction (namely \( \frac{m}{q} \)).

This is a correct proof.

\[ \text{A} \]

\[ \text{B} \]

\[ \text{C} \]

\[ \text{D} \]

50 Proposition: The quotient of a true fraction and a natural number is a true fraction.

Proof: Suppose the quotient of a true fraction \( \frac{a}{b} \) and a natural number \( c \) is the natural number \( n \), then \( \frac{a}{bc} = n \).

If \( \frac{a}{bc} = n \), then \( \frac{a}{b} = \frac{1}{cn} \).

But \( c \) and \( n \) are both natural numbers, therefore \( cn \) is a true fraction.

This is a correct proof.

\[ \text{A} \]

\[ \text{B} \]

\[ \text{C} \]

\[ \text{D} \]

51 Proposition: If the square root of a true fraction is a fraction, then it is a true fraction.

Proof: Suppose the square root of the true fraction \( \frac{a}{b} \) is a fraction, and is a natural number \( n \), then

\( \sqrt{\frac{a}{b}} = n \).

If \( \sqrt{\frac{a}{b}} = n \), then \( \frac{a}{b} = n^2 \).

As \( n \) is a natural number, then \( n^2 \) is a natural number, and the statement \( \frac{a}{b} = n^2 \) contradicts that \( \frac{a}{b} \) is a true fraction.

Therefore \( \sqrt{\frac{a}{b}} \) cannot be a natural number. If it is a fraction, then it must be a true fraction.

This is a correct proof.

\[ \text{A} \]

\[ \text{B} \]

\[ \text{C} \]

\[ \text{D} \]
The problem is to devise a method of inscribing a square inside a triangle: two vertices of the square should be on the base of the triangle, and the two other vertices of the square should be on the two other sides of the triangle, one on each. A student makes a few tentative sketches with three vertices in the correct position, as shown on the right.

In order to solve the problem, which of the following is the most likely to lead to an elegant solution?

A. continue increasing the side of the square till it is exactly right
B. redraw the triangle $OMN$ to make it equilateral
C. try to show that the points $O, P, Q, R$ lie on a straight line
D. try a new approach using rectangles instead of squares

Questions 53 to 56 refer to the following information:
A number of medical insurance associations provide refunds on payments made for medical treatment and hospitalization, the extent of the refund depending on the premium paid. The table below gives details of refunds obtained for three different premiums.

<table>
<thead>
<tr>
<th>Premium</th>
<th>Hospital refund per week</th>
<th>Medical refund per visit to surgery</th>
<th>Refund for visit to specialist</th>
</tr>
</thead>
<tbody>
<tr>
<td>P, $62.00 p.a.</td>
<td>$66.50</td>
<td>$1.80</td>
<td>$5.00</td>
</tr>
<tr>
<td>Q, $52.00 p.a.</td>
<td>$63.00</td>
<td>$1.60</td>
<td>$3.00</td>
</tr>
<tr>
<td>R, $48.00 p.a.</td>
<td>$56.00</td>
<td>$1.40</td>
<td>$4.00</td>
</tr>
</tbody>
</table>

Refund for treatment $X$—$25.00
Refund for treatment $Y$—$40.00
Refund for treatment $Z$—$50.00

Premiums P, Q, or R.

53 A contributor to premium R makes two visits to a specialist (fee $10 per visit) after three visits to surgery (fee $4 per visit). If we ignore the cost of the premium, the excess of his payments over his refunds would be (to the nearest dollar)

A. $14
B. $24
C. $34
D. $20

54 A contributor to premium P spends 9 days in hospital (fee $84 per week) for treatment $Z$ (fee $70). His expenses for hospitalization and treatment are greater than his refund by an amount of

A. $25.50
B. $42.50
C. $65.50
D. $37.00

55 A contributor has three visits to a general practitioner (fee $3 per visit) followed by two visits to a specialist (fee $10 per visit), then treatment $X$ (fee $40.00). The treatment involved hospitalization for three days (fee $12 per day). This contributor was on premium Q. The difference between the total fees he paid and the refunds he obtained was (to the nearest dollar)

A. $38
B. $42
C. $26
D. $54

56 For one year a family's total medical bills resulted from 30 visits to a general practitioner (fee $3.00 per visit). The most economical action for this family for that year would have been

A. to pay premium P
B. to pay premium Q
C. to pay premium R
D. to pay none of the above premiums.
57 Which of the following statements about the number
\[(2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9) + 1\]
is, from inspection, certainly correct?
A It is a prime number.  
B It is an even number.  
C It is not divisible by 9.  
D It is not divisible by 23.

58 The boys in a certain class have an average height of 5 ft 5 ins and the girls in the class have an average height of 5 ft 2 ins. You are required to find the average height of the children in the class. Which of the following is correct?
A Just enough information has been given to solve the problem.
B One more piece of information is sufficient to solve the problem: namely, the fraction of the class consisting of boys.
C Two more pieces of information are necessary before the problem can be solved: namely, the number of boys in the class, and the number of girls in the class.
D A surplus of information has been given.

Questions 59 and 60 refer to the following information:
Consider the operator \( \pi \) such that
\[\prod_{i=1}^{n} X_i = X_1 \cdot X_2 \cdot X_3 \cdot \ldots \cdot X_n\]
So \[\prod_{i=1}^{3} X_i = X_1 \cdot X_2\] and \[\prod_{i=1}^{4} X_i = X_1 \cdot X_2 \cdot X_3 \cdot X_4\]

59 The values of \( X_i \) represent the consecutive terms of a sequence, the first four values of which are given below:
1, 3, 5, 7, ...

The value of \[\prod_{i=1}^{5} X_i\] is
A 9430  
B 945  
C 105  
D 27

60 The value of \[\prod_{i=2}^{3} (1 + X_i)\] is
A \((1 + X_2) (1 + X_3) (1 + X_4)\)  
B \(1 + X_1 + X_2 + X_3\)  
C \(3X_1 \cdot X_2 \cdot X_3\)  
D \(1 + X_1 \cdot X_2 \cdot X_3\)

61 All Saints’ Day is always on the first day of November; in 1968 this was Friday. The Melbourne Cup (a race) is always run on the first Tuesday in November. When will the next Melbourne Cup be run on All Saints’ Day?
A 1972  
B 1973  
C 1977  
D Never; they wouldn’t dare to have racing on a Saint’s Day.


A. C. Brookes, Government Printer, Melbourne.