7-30-1970

Quantitative Thinking 1970

ACER

Multi - Aptitude Watters - 26 (5)

30

commonwealth
secondary
scholarships
examination

morning session:
thursday
30 july
1970

time allowed:
two hours

test booklet
to be handed
in with
your answer
sheet

quantitative thinking

instructions to candidates

This is a test of your ability to use basic mathematical principles and ideas. If a question involves terms or principles which you have not met before, these will be explained in sufficient detail to enable you to answer the question. You may also want to make use of the symbols and formulae printed on this page.

You will obtain the best possible score if you observe the following points:
(1) Work carefully through the questions in the order in which they are given.
(2) Don't waste too much time on any one question; if necessary, go on to the next question and come back to the difficult ones later. (3) If you think you know an answer, mark it—even if you are not certain you are correct. Marks will **not** be deducted for wrong answers.
(4) Make sure that you mark the letter you have chosen in the correct line on your answer sheet.

answering

Each question (with two exceptions) has four alternative answers, represented by the letters A B C D. You must choose one answer from these alternatives. Having done so, make sure you mark your answer correctly.
Questions 22 and 23 are the exceptions; they have only three alternative answers to choose from.

If you want to change an answer, erase your first mark completely. Try to avoid having to make erasures by not answering hastily. Take care that your pencil mark does **not cross** into another row or column, and that there are no marks or smudges on your answer sheet.
Now look through this booklet, but **don't start writing** until the supervisor tells you to do so.

symbols

$=$ **means 'is equal to'**
$\neq$ **means 'is not equal to'**
$>$ **means 'is greater than'**
$\geqslant$ **means 'is greater than or equal to'**
$<$ **means 'is less than'**
$\leqslant$ **means 'is less than or equal to'**
$\llcorner$ **indicates that the angle between the two lines is a right angle**
$\angle$ **ABC means 'angle ABC'**
$\triangle$ **ABC means 'triangle ABC'**
$\fallingdotseq$ **indicates that the two lines are parallel**
$\eqslantless$ **indicates that the two lines are equal**

formulae

circumference of a circle
$=$ $2\pi \times$ radius, i.e. $C = 2\pi r$
area of a circle
$=$ $\pi \times$ square of radius, i.e. $A = \pi r^2$
area of a rectangle
$=$ length $\times$ breadth, i.e. $A = l \times b$
area of a triangle
$=$ $\frac{1}{2} \times$ base $\times$ height, i.e. $A = \frac{1}{2}b \times h$

Questions 1 and 2 refer to the following information:

It can be proved (by checking) that

$$1^3 + 2^3 = 3^2,$$

and $1^3 + 2^3 + 3^3 = 6^2$

1 If we were able to generalize from these two statements, we would expect $1^3 + 2^3 + 3^3 + 4^3$ to be equal to

A 9^2 **C** 10^2

B 9^3 **D** 10^3

2 Again, if we were able to generalize, how many such numbers, each cubed, would add together to give a total of 441?

A 4 **C** 6

B 5 **D** 7

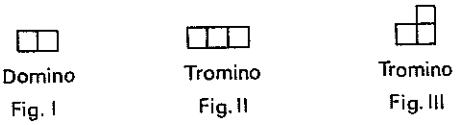
3 If $\frac{x}{3} = \frac{y}{4}$, what is the value of xy?

A 12 only **C** $\frac{3}{4}$ only

B $\frac{4}{3}$ only **D** It may have many values.

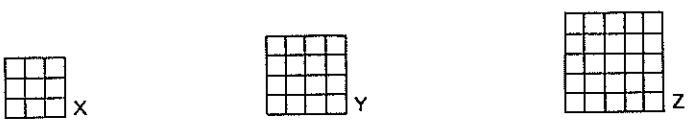
Questions 4 to 7 refer to the following information:

A polyomino is defined as a 'simply connected' set of squares, i.e. a set of squares which are joined along their edges. There is only one type of domino (2 connected squares) and there are two types of trominoes (3 connected squares). Tetrominoes have 4 connected squares.



Domino Fig. I | Tromino Fig. II | Tromino Fig. III

Assume that a large number of dominoes, trominoes and tetrominoes of all types are available. Below, three square boards are illustrated.



Use the following key to answer Questions 4 and 5:

KEY

A X, Y and Z

B Y only

C X only

D none of the boards

4 Which of the square boards may be completely covered, without overlap, using only trominoes? (Answer **A**, **B**, **C** or **D**.)

5 Which of the square boards may be completely covered, without overlap, using only tetrominoes? (Answer **A**, **B**, **C** or **D**.)

6 It is desired to cover a 6×5 board (see Fig. IV) from the available polyominoes. What is the smallest number of polyominoes required?

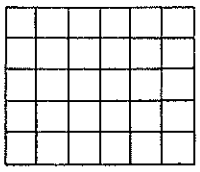


Fig.IV

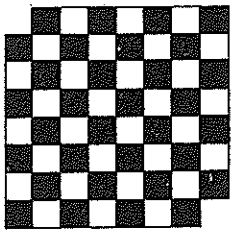
A 8

B 9

C 10

D It is not possible to satisfy the given conditions.

7 Two squares from opposite corners of a chess board are removed as in Fig. V. Is it possible to cover all squares of the board so produced with dominoes?



FigV

A no, because the board is no longer a perfect square

B yes, because the board is still symmetrical

C yes, because there still remain 62 squares which may obviously be covered by 31 dominoes

D no, because each domino must cover a black square and a white square and there are now more black squares than white squares

Questions 8 and 9 refer to the following information:

In a game, considered by some to be the greatest game of all, a team is awarded six points if they kick a 'goal' and one point if they kick a 'behind'. p is the number of points scored by a team who kick g goals and b behinds.

8 Which of the following could be used to calculate p?

A $6(g + b) = p$

B $\frac{g}{6} + b = p$

C $6g + b = p$

D $g + 6b = p$

9 What is the smallest number of behinds a team can kick, to score exactly 100 points?

A 1
B 4
C 6
D 10

Questions 10 to 12 refer to the following tables and information:

People are watery animals. About half of our total weight is just water. The table gives more details.

Weight of Water as a Percentage of Total Body Weight

Age	Male	Female
10–18 ..	59%	57%
19–40 ..	61%	51%
41–60 ..	55%	47%
Over 60 ..	52%	46%

10 Which entry in the table seems to be contrary to the general trend of the change in water content of the body with age?

A male 19–40
B female 19–40
C male 41–60
D female 41–60

11 At which of the following ages would you expect there to be **least** difference in the percentage of water in the body between males and females?

A 65
B 45
C 22
D 8

12 A student claims that for all people aged over 60 the average percentage weight of water in the body is 49 per cent.

A This claim is justified.
B This claim is justified only if there are as many men as women over the age of 60 in the population.
C This claim is justified if the average weight of men and women in this age range is the same.
D This claim directly contradicts the information given.

Questions 13 to 15 refer to the following information:

The digital sum of a number is obtained as demonstrated in the following example:

the digital sum of 27,233 is 8 because $2 + 7 + 2 + 3 + 3 = 17$ and $1 + 7 = 8$

A natural number greater than 1 is called a perfect number when the sum of its divisors is twice the number. For example, 28 is a perfect number because its divisors are 1, 2, 4, 7, 14 and 28, and the sum of these divisors is 56.

13 What is the digital sum of 1905?

A 4 **C** 7

B 6 **D** 15

14 What is the smallest perfect number?

A 2 **C** 12

B 6 **D** 28

15 The digital sum of 28 is 1. The digital sums of each of the next few perfect numbers is also 1. One of the following will be the next perfect number after 28. Which is it?

A 37 **C** 496

B 235 **D** 817

Question 16 refers to the following information:

A regular hexagon is a six-sided figure with all sides equal and all internal angles equal.

16 $PQRSTU$ is a regular hexagon. The areas of the triangles PQR and URS are w and y respectively. The area of the hexagon $PQRSTU$ is equal to

A $2(w + y)$ **C** $(w + y)$

B $4(w + y)$ **D** $\frac{3}{2}(w + y)$

Questions 17 and 18 refer to the following information and key:

The following two questions consist of a pair of measurements. Your task is to compare the two elements that make up the pair. For each pair give the description in the key which is most appropriate.

KEY
- **A** The first element of the pair is greater than the second element.
- **B** The second element of the pair is greater than the first element.
- **C** The two elements are not comparable.
- **D** The two elements are comparable, but they are not sufficiently known to make a comparison.

17 your height and your weight

18 the chest measurement of a boy and of his father

19 How many integers (whole numbers) are there between 101 and 999 that are exactly divisible by 50?

A 17 **C** 19

B 18 **D** none of the above

Questions 20 and 21 refer to the following information:

A regular hexagon is a six-sided figure with all sides equal and all internal angles equal.

20 What is the **least** number of triangles that could be fitted together to form a regular hexagon?

A 3
B 4
C 5
D 6

21 What is the **least** number of identical triangles that could be fitted together to form a regular hexagon?

A 3
B 4
C 5
D 6

Questions 22 and 23 refer to the following key:

KEY **A** The statement is true.
B The statement is false.
C It is not possible to determine whether the statement is true or false.

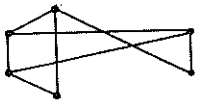
22 June can have five Sundays and five Mondays in it.

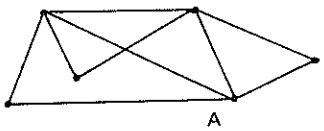
23 Any two days of the week can occur five times in the same month provided it is leap year.

Questions 24 to 26 refer to the following information:

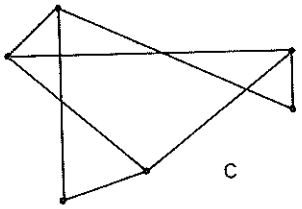
The following diagrams depict patterns composed of lines joining particular points. All the points may be moved around in space but no more than one point may occupy the same position at any one time. The lines may stretch or contract.

24 Which one of the following patterns (**A**, **B** or **C**) could be obtained from the pattern on the right? If none of them, write **D**.

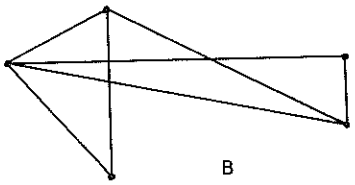




A

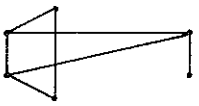


C

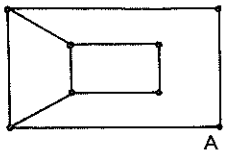


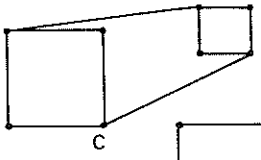
B

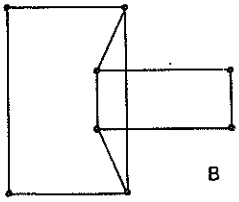
25 Which one of the patterns (**A, B** or **C** in Question 24) could be obtained from the pattern on the right? If none of them, write **D.**

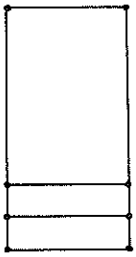


26 Which one of the following patterns could **not** be obtained from the others?


A


C


B


D

Questions 27 to 29 refer to the following information:

x, y, z and w represent four numbers. Three of the numbers are equal to unity (one) and the other is equal to zero.

27 The expression $(x + y) \times (z - w)$

A must be equal to 1 or 0.
B must be equal to 2 or 0.
C must be equal to 2 or -2.
D must be equal to 2 or -2 or 0.

28 The expression $x + y - z - w$

A must be equal to 0.
B must be equal to 1.
C must be equal to 1 or -1.
D must be equal to 1 or -1 or 0.

29 The expression $\dfrac{x - y}{z + w}$

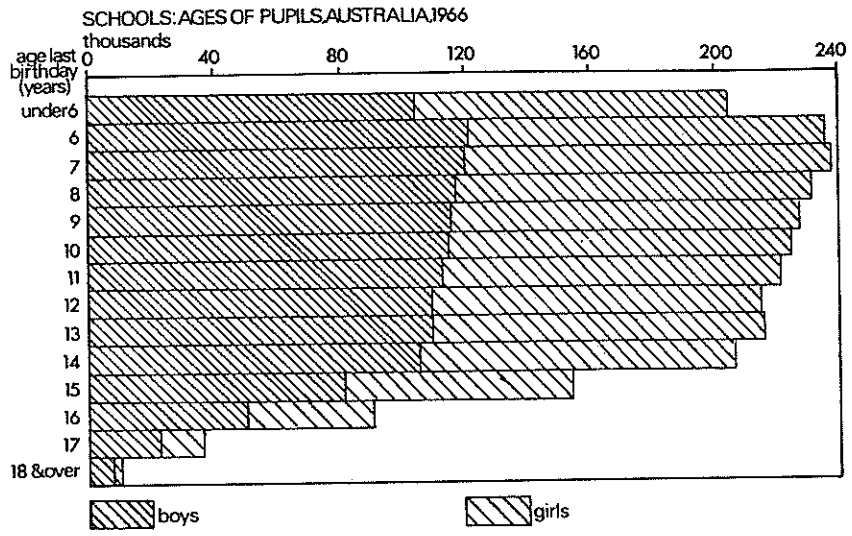
A must be equal to 0.
B must be equal to $\frac{1}{2}$.
C must be equal to $\frac{1}{2}$ or $-\frac{1}{2}$.
D must be equal to 0 or $\frac{1}{2}$ or $-\frac{1}{2}$.

30 Which of the following is closest to the value of

$$\sqrt{\frac{1.234 \times 98.31 \times 10003}{813.4 \times 0.34 \times 10^2}} \ ?$$

A 10
B 150
C 700
D 5,000

Questions 31 to 34 refer to the following graph:



31 Of all children at school from the ages of 8 to 13,

A there are more boys than girls.

B there are more girls than boys.

C there is an equal number of boys and girls.

D there are probably different numbers of boys and girls, but there is insufficient information to decide which is greater.

32 If we consider the age range from 7 to 14, the number of children at school decreases as the age increases. The most probable explanation of this is that

A children are continually leaving school.

B the total population is continually increasing.

C children are continually arriving at school.

D the total population is continually decreasing.

33 Of the 15-year-olds at school in 1966, what percentage is likely to be at school in 1967?

A 40 per cent

B 50 per cent

C 60 per cent

D 75 per cent

34 Which of the following is the best estimate of the total number of children in school in Australia in 1966?

A 240,000

B 2,500,000

C 1,400,000

D 3,500,000

Questions 35 to 38 refer to the following information:

A set of elements is all of the same form xM where x is any number and the sort of thing that M is does not matter. The rule for combining two elements xM and yM in this set is given by $xM * yM = 2xyM$.

For example $3M * 2M = 12M$

and $5M * -3M = -30M$

35 If $xM * 3M = 2M$, then x is equal to

A $\frac{1}{3}$

B $\frac{3}{2}$

C $\frac{2}{3}$

D none of these

36 If $xM * yM = yM * xM$, then

A $x = y$

B $x < y$

C $x > y$

D the relationship between x and y cannot be determined from the information given

37 If the result of combining the element eM with any element xM is xM, then the value of e is

A 0

B $\frac{1}{2}$

C 1

D dependent on the value of x

38 If $xM * xM = x^2M$, then the value of x is

A 0

B 1

C -1

D undecidable from the information given

Questions 39 to 42 refer to the following information:

A sequence of square numbers is built up as follows:

a. The first square number is 1, represented—

.

b. The second square number is 4, represented by a unit square—

. .
. .

c. The third square number is 9, represented by a square which is built from a number of unit squares, as indicated—

. . .
. . .
. . .

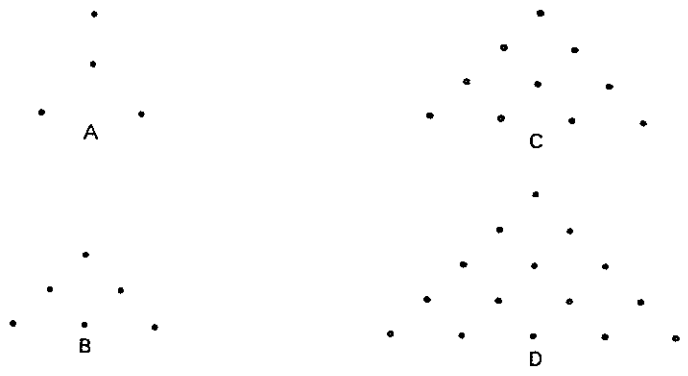
39 The fifth square number is

A 30
B 25
C 15
D 16

Questions 40 to 42:

Assume that sequences of triangular, hexagonal and cubical numbers are built up in a similar manner to the sequence of square numbers.

40 Which of the following represents the fourth triangular number?



41 How many dots would you need to make the third hexagonal number?

A 6
B 7
C 10
D There is no third hexagonal number: there are only two.

42 How many dots would you need to make the fourth cubical number?

A 64 **C** 16

B 32 **D** 8

43 A precious stone weighing 25 carats broke into two pieces. When the two pieces were put on a balance, one on either side, an additional weight of 9 carats was required to maintain balance. The larger of the two pieces weighed

A 17 carats. **C** $21\frac{1}{2}$ carats.

B 14 carats. **D** 19 carats.

In Questions 44 to 46 you are asked a question, and then given two additional pieces of information, (a) and (b). Use the following key to answer Questions 44 to 46.

KEY

A One only of (a) or (b) is sufficient to answer the question. The other is not sufficient.

B (a) and (b) together are sufficient to answer the question but neither of them alone is sufficient.

C Each of the two pieces of information (a) and (b) is alone sufficient to answer the question.

D More information is needed to answer the question.

44 Is $p > q$?

(*a*) $q < r$

(*b*) $r \leqslant p$

45 Is the curve linking A to B straight?

(a) AB is the shortest distance between A and B on a plane.

(b) The slope of AB is constant.

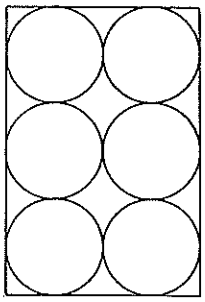
46 Is $ABCD$ a square?

(a) $ABCD$ is a closed figure with 4 straight sides, having AB equal to and parallel to DC.

(b) $\angle ABC = 90°$

Questions 47 and 48 refer to the following diagram and information:

A rectangular box has inside measurements 8 inches by 12 inches by 5 inches high. Six cylindrical tins each of diameter 4 inches and height 5 inches are placed in the box as shown.



47 What area of the base (in square inches) is left uncovered by the tins?

A 24π

B 96π

C $96\,(1 - \pi)$

D $24\,(4 - \pi)$

48 What is the value of the ratio $\dfrac{\text{volume taken by tins}}{\text{total volume of box}}$?

A $\dfrac{1}{\pi}$

B $\dfrac{4}{\pi}$

C $\dfrac{\pi}{4}$

D $\dfrac{4\pi}{20 - \pi}$

49 If x cents is equivalent to y dollars, what is the value of y in terms of x?

A $100x$

B $\dfrac{x}{100}$

C 100

D $\dfrac{100}{x}$

50 ABC is a triangle, in which $AD = \frac{1}{2}AB$, and $AE = \frac{1}{2}AC$.

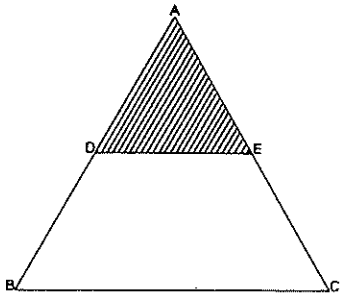
If ABC is an equilateral triangle, then the area of the triangle ADE is

A $\frac{1}{4}$ the area of $\triangle ABC$.

B $\frac{1}{3}$ the area of $\triangle ABC$.

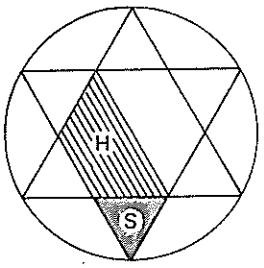
C $\frac{1}{2}$ the area of $\triangle ABC$.

D none of the above.



51 A regular star is illustrated on the right.
What is the value of the ratio

$$\frac{\text{number of square units in hatched area } (H)}{\text{number of square units in shaded area } (S)}\ ?$$



A $\frac{3}{2}$

B 2

C 3

D $\frac{\pi}{3}$

52 x and y are digits, and the number $xxyy$ is a four-digit number. Which of the following numbers will always divide evenly into this four-digit number?

A 11 **C** y

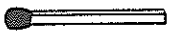
B 101 **D** xy

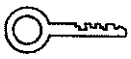
Questions 53 and 54 refer to the following information:

Two surfaces are said to be topologically equivalent if each of them can be transformed into the other by stretching, but without cutting or tearing. Assume that the solids referred to in Question 53 and Question 54 can be stretched or contracted in any direction without tearing.

53 Which one of the following solids has a surface that is **not** topologically equivalent to the surface of a drinking straw?

A flowerpot 

B match 

C key 

D chainlink 

54 Which of the following has surfaces that are topologically equivalent to the surfaces of three consecutive links in a chain?

A tyre, tube and bicycle wheel

B bolt, washer and nut

C two quoits and the peg on which they are thrown

D none of the above

55 To the nearest thousandth, $\sqrt{2} = 1.414$. You are required to find the value of $\sqrt{2}\,(10 + \sqrt{2})$. Which of the following is the best way to proceed?

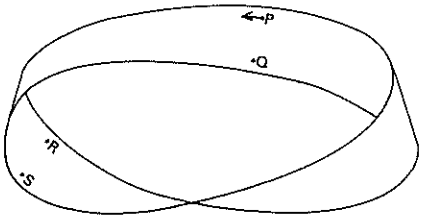
A Calculate the section inside the bracket and multiply by $\sqrt{2}$.

B Realize that 10 is $\sqrt{100}$ and include the 100 and the 2 under the root sign.

C Divide each term by $\sqrt{2}$; then do the calculation, then multiply each term by $\sqrt{2}$.

D Multiply each term in the bracket separately by $\sqrt{2}$ and then add.

56 The diagram below shows a twisted strip of opaque paper of width 1 inch. P, Q, R and S are dots, each of which is $\frac{1}{4}$ inch from the edge of the paper **and marked on one side only.** A pencil line is drawn from P in the direction shown keeping $\frac{1}{4}$ inch from the edge of the paper.



In which order will the pencil pass through some or all of the points?

A *PSQRP*

B *PSPSP*

C *PRPRP*

D *PRQSP*

57 In a scientific experiment the relationship between two variables is being studied. The product of two variables is K where K is a positive constant. If one variable is negative and decreasing, the other must be

A negative and increasing.

B negative and decreasing.

C positive and increasing.

D positive and decreasing.

58 A number is to be identified according to the set of conditions (i), (ii), (iii) given below. You may ask additional questions which can be answered 'yes' or 'no'. What is the least number of questions you must ask to be sure of identifying the number?

(i) It is the square of a whole number.

(ii) It is less than 150.

(iii) Its final digit is 6.

The least number of questions to identify the number is

A 0

B 1

C 2

D 3

59 A hexagon is a six-sided figure. A number of hexagons are drawn, so that each side of each hexagon is 1 inch in length. These hexagons

A must all be identical.

B could be of two possible shapes.

C could be of three possible shapes.

D could be of a large variety of possible shapes.

Questions 60 and 61 refer to the following information and diagram:

If the shape shown in Fig. I is cut out and folded, it forms a solid like the one shown in Fig. II.

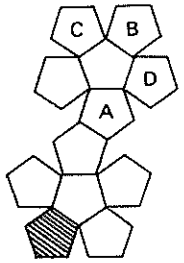


Fig. I

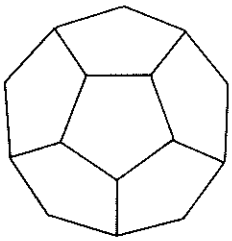


Fig. II

60 Which of the lettered faces **(A, B, C** or **D)** would then be parallel to the face which is hatched?

61 Which faces have an edge common to the hatched face?

A *B* only

B *C* only

C both *B* and *C*

D neither *B* nor *C*

Prepared and published by Australian Council for Educational Research, Frederick Street, Hawthorn, Victoria 3122; June 1970.

Printed for the Government of the Commonwealth by W. G. MURRAY, Government Printer, Canberra.