Mathematical mindsets: Fostering student engagement and positive mindsets through the use of challenging tasks

Alison Hall
Australian Council for Educational Research

https://doi.org/10.37517/978-1-74286-715-1-12

Alison Hall joined ACER in 2022 and is a researcher in the Primary, Early Childhood and Inclusive research team where she works predominantly in test development in early years numeracy. She is also working on projects such as the Digital Assessment Library for VCAA, item writing in mathematics for the NSW Department of Education, analysing learning trajectories for the Australian Education Research Organisation (AERO) and researching mathematics foundation skills for the UN Sustainable Development SDG 4.2 LP Pre-school Progression Project. Alison has a background as a generalist primary teacher and as a Mathematics Leader, teaching classes from years F to 6. She has also completed a Master of Education in Mathematics Leadership.

This paper will be presented by Greta Rollo from ACER on behalf of Alison Hall.

Abstract

This paper explores the planned use of challenging mathematical tasks. These tasks provide the opportunity for students to improve mathematical thinking by working on problems that they do not yet know how to answer. This research involved a heterogeneous class of year 3 students from a Catholic parish primary school in the northern suburbs of Melbourne. A rubric was also developed to use in conjunction with these tasks, to support discussions with students, broaden their strategies in finding solutions and thereby improve their conceptual understanding. These pedagogical approaches were found to support the improvement of both students' conceptual understanding in mathematics and teachers' reflective practice.

Introduction

This paper examines action research that tested a rubric as a method of identifying different levels of thinking by students working on challenging mathematical tasks. The research explored ways of using challenging tasks to increase the sophistication of students' mathematical strategies and explanations. The study aimed to answer the question: 'To what extent can rubrics be used to support teachers' use of challenging tasks to broaden the sophistication of students' mathematical concepts?'

This study involved the development and use of a rubric as an instrument to support teachers, students and other stakeholders in knowing where students are in their mathematical learning and 'where to next'. The use of rubrics may allow teachers to infer gaps between a student's existing learning and the learning objectives. Francisco and Maher (2011) find that observations made by teachers helps them realise the value of providing students with opportunities to explore ideas and make decisions about their own mathematical reasoning and its development. Teachers need to know how to assess student's reasoning, in addition to assessing mathematical skills, but they must be deliberate in the choice of tasks that ask students to apply both reasoning and mathematical skills.

Research evidence supports the use of challenging tasks in developing students' mathematical reasoning skills. Clark and Clark (2002) recognise 4 qualities that characterise such tasks: they must enable students to produce different solutions; use different strategies; offer diverse final presentations; and fully engage students. Boaler (2019) argues that number sense, which is fundamentally important for students to learn, includes the learning of mathematical facts along with a deep understanding of numbers and the ways they relate to each other. Cheeseman et al. (2013) reflect on practices teachers use with these tasks and identify that students can engage with important mathematical ideas, be encouraged to explain their strategies, justify their thinking, and extend their knowledge in new ways.

Teachers must also consider a range of pedagogical factors when selecting challenging tasks. Kirschner et al. (2006) advocate for guided instruction being superior to minimal or even no guidance with challenging tasks. However, Marshall and Horton, (2011) alongside Russo and Hopkins (2017) agree that the value of exploring concepts, before any direct instruction, is in realising students' abilities to reason and think critically. Sullivan et al. (2012) also acknowledge the importance of matching tasks to curriculum content when selecting tasks. Cheeseman et al. (2016) discuss the importance of how a teacher introduces a task, including preparing students to have persistence, connecting the task with student experiences, providing manipulatives, and clarifying the task without showing how to reach the solution. This paper explores an approach aligned with the views of Russo and Hopkins.

Choosing tasks, structuring lessons around them and then incorporating them successfully into a thematic program of work requires careful thought. Teachers may need support in developing a classroom culture that supports this style of learning (Sullivan et al., 2013). Teachers can, therefore, be disinclined to use challenging tasks (Cheeseman et al., 2013) because they view them as unclear, too demanding or are concerned about low-attaining students. Clarke et al. (2014) highlight the significance of a teacher's interest in a task in influencing its success, as well as teacher confidence in the enthusiasm and ability of their students. Kirschner et al. (2006) suggest that minimising instruction may lead to misunderstandings or piecemeal knowledge, so a teacher's approach needs to be balanced against providing too much information to reduce the level of challenge within the task. Jacobs et al. (2014) notes the dangers of teachers taking over student thinking, controlling available tools and asking closed questions, and removing the agency of students in their learning and development of conceptual understanding. Simon (2017) argues that an understanding of mathematical concepts requires students to learn concepts through mathematical activities in the form of challenging tasks. Rather than students using a sequence of actions already available to them based on their prior knowledge, challenging mathematical tasks support students to build new knowledge. The current study looks at supporting teachers to give sufficient, but not excessive instruction, using an assessment rubric to provide appropriate, timely feedback to students that progresses their conceptual mathematical understanding.

The rubric developed for this study (see Appendix A) drew from Bloom's Taxonomy (Krathwohl, 2002) and Webb's Depth of Knowledge Framework (DoK) (1997). Webb suggests that 'challenge' in learning tasks promotes growth by keeping students engaged and his framework describes the quality of student thinking in various tasks. Krathwohl's update of Bloom's Taxonomy describes the cognitive level students demonstrate during learning, while the DoK focuses more on the context – in this case the challenging task. While Hess et al. (2009) identify some limitations with Bloom's Taxonomy, the current study incorporates both Bloom's and Webb's models into the rubric.

Project Design

Participants

The participants were members of a heterogeneous year 3 classroom in a Catholic parish primary school in the northern part of Melbourne. The researcher was the full-time teacher of the class. A preservice teacher was also working full time in the classroom at the time of the study and was involved in the data collection. There was no requirement for a selection process as a convenience sample was used.

Method and rationale

This study used an instrumental case study approach alongside action research. The action research aspect addressed the need to improve practices and the instrumental case study approach aligned with the observation of a situation. Such approaches provide teachers with opportunities to apply research methods to their teaching (Mills et al., 2010). They can also improve teachers' understanding of classroom practices and raise awareness of student learning that requires further investigation. Teachers can test approaches that may transfer well to similar classrooms (Yin, 2014) and integrate assessments generated by their own research into practice.

The current study used 2 tasks designed by Russo. These were Task 1, The Doughnut Tree task, which explores exponential doubling (Russo, 2006a); and Task 2, The Big (not so) Friendly Giant task, which explores halving (Russo, 2006b). Task 1 was chosen because the class had been working on multiplication using doubling. Russo (2016a) contends that students working in middle primary classrooms are expected to have developed fluency with their doubles facts and should be exploring doubling as a rule. He argues that students would benefit from exploring exponential doubling at a younger age. Task 2 was chosen to meet the needs of a diverse group of students, providing both accessibility and extension using enabling and extending prompts. Both tasks address the mathematically related skill of doubling and its inverse, halving, supporting students to link these 2 concepts. These tasks were conducted and data were collected from one class in term 3 of 2018.

Enabling prompts (see Appendix B) are an integral aspect of challenging task design as they reduce the level of challenge through simplifying the problem, changing how the problem is represented, helping the students connect the problem to prior learning and/or removing a step in the problem (Sullivan et al., 2006). Extending prompts can be used to engage students who finish the main task and may expose students to an additional task that is more challenging, but still requires them to use similar mathematical reasoning, conceptualisations and representations as the main task (Sullivan et al., 2006). The appropriate prompt is selected by the teacher in real time, developed from their analysis of the potential task difficulties based on perceived cognitive load.

Russo and Hopkins (2017), reflecting on cognitive load theory (Sweller, 1998), identify 7 steps to produce challenging mathematical tasks that aim to optimise the cognitive load for each student. These steps are: identify the primary learning objective, develop the task, look for possible other learning objectives, sort any objectives in line with their cognitive load, redesign the task, develop prompts to optimise the cognitive load and propose a lesson summary.

A launch, explore, discuss model (Stein et al., 2008) was used to deliver the lessons. This facilitated more explicit explanations and scaffolded connections, and highlighted big mathematical ideas. In the launch phase, the word 'challenging' was discussed with the students and the word was defined to engage the students in characterising an appropriate mindset. Appendix C provides examples of student's verbal responses. Each problem was introduced in a separate lesson, alongside available materials and recording expectations.

In the explore phase, students worked on the task individually or in pairs. Students were supported in solving the problem in whichever way suited them. Enabling prompts were offered and students had access to counters, number lines, 100 squares and notes about doubling and halving, and were able to ask clarifying questions. While students were working on the solutions, 3 main questions were asked: How would you describe the problem in your own words? Would it help to create a diagram, draw a picture or make a table? Could you try it with different numbers?

In the discuss phase, the teacher presented a summary of what had been observed, referring to specific strategies used by students, some of whom shared their thinking with the class. After looking at work from Task 1, it was noted that although students were solving the problem, their thinking was not clearly shown. Support to assist this was provided in the launch phase for Task 2. Appendix D provides examples of student verbal responses to questioning by the preservice teacher.

In response to observations and discussion with the preservice teacher, a follow-up lesson was proposed based on discussing ways of presenting strategies and solutions, answering questions with detail, revising, editing work and explaining mathematical reasoning. It was felt that the students required more explicit teaching. alongside detailed examples of possible ways of presenting their solutions and reasoning. The lesson was based on a simpler task 'What else belongs' (see Appendix E).

Student work

Student work was assessed against criteria from the rubric rather than being competitively ranked. This approach provides students with feedback regarding how to improve rather than how they compare with others. A process of moderation to provide inter-relater reliability was undertaken. Work was de-identified by the preservice teacher and shared between a team of 3: the researcher, the numeracy leader and the preservice teacher. Any work pieces with uncertain scoring were grouped for further consideration by another member of the team. If there was still uncertainty, the whole team would look at the work. For the purposes of moderation, each team member selected a sample piece for each level of thinking using the DoK stages, and these were compared. For this study, due to ethical requirements, no student work could be reported or presented.

Limitations of the data collection

Student work was collected at the end of each teaching session by the preservice teacher to maintain as much anonymity as possible. Apart from recognising some handwriting the researcher was not aware as to who had completed which work samples. According to Fraser (1997) the concept of a teacher as a researcher enables credible educational research to be undertaken, but the ethical predicaments faced could be more challenging than those met by an external researcher.

Data sources and analysis

The primary data for analysis were generated by scoring student performances using rubrics and was based on expert teacher evaluation of student responses and explanations of their thinking during challenging tasks and student work (artefacts). Artefacts were grouped in terms of similar approaches to the task. These were rated against the rubrics. Scores were then categorised according to the nature of the students' responses. The final analysis involved a final sample size of 15 students selected randomly from those who had completed both tasks.

The approaches used by students reflected varying degrees of sophistication in their application of mathematical strategies. Approaches included the use of drawings, number lines, repeated addition and subtraction, formal algorithms, partitioning numbers to make doubling/halving simple, as well as the direct use of multiplication and division. The use of drawings was the most common strategy followed by partitioning numbers. The use of number lines and repeated addition and subtraction occurred with equal frequency. The direct use of multiplication facts and division was rare in both tasks.

Analysis

Table 1 shows the types of thinking – Problem-solving, Reasoning, Representation and Connection – with a zone of proximal development (ZPD) (Vygotsky & Cole, n.d.) for each task. It appears that in Task 1, students had more problems with Representation whereas Connection was more of an issue in Task 2. Scores for 3 areas – Problem-solving, Reasoning and Representation – increased quite considerably, whereas the Connection score remained very similar. It appears this was an area with which many students struggled and where future explicit teaching needs to be focused. Problem-solving scores had the greatest increase. This could be attributed to greater familiarity with the type of tasks, students acting on discussions and feedback from the Task 1 and/or students finding Task 2 easier to solve.

Table 1 Conditional formatting of rubric elements for analysed work samples to create a ZPD

	Task 1 The Doughnut Tree				**Task 2 The Big (not so) Friendly Giant**			
	Problem-solving	**Reasoning**	**Representation**	**Connection**	**Problem-solving**	**Reasoning**	**Representation**	**Connection**
22	1	1	1	1	1	1	1	1
16	1	1	1	1	2	2	2	1
19	1	2	1	1	2	2	2	1
23	1	2	2	2	2	1	1	1
21	1	1	1	1	3	3	3	1
5	1	1	1	2	3	3	2	1
27	2	2	2	2	2	2	2	1
12	2	2	1	1	3	3	3	1
6	2	2	1	1	4	3	2	2
10	2	2	2	2	4	3	2	1
8	2	2	2	2	3	3	3	2
26	2	2	2	2	3	3	3	2
28	3	2	2	2	3	3	3	3
11	3	2	2	2	4	3	4	4
24	3	3	2	1	4	4	4	3

Results and evaluation

The data analysis and results were related back to the initial research question: 'To what extent can rubrics be used to support teachers' use of challenging tasks to broaden the sophistication of students' mathematical concepts?' Concepts analysed included Problem-solving, Representation and Reasoning. Results from the ZPD analysis showed several interesting findings. Nearly all students used a broader range of strategies in the Task 2 compared with Task 1. Students who used broader strategies used them at a more sophisticated level. Most students diversified the way they represented their thinking. About half of the students showed limited development in Connection, indicating an area for targeting teaching strategies. That students were least successful with Connection confirmed the teachers' view that in previous tasks where students focused on solving the problem rather than conceptual understanding. The element of Connection scored low in both tasks. This may have been because these tasks were different from other mathematics tasks undertaken and students viewed them in isolation from their everyday mathematics lessons.

Conclusion

This research was undertaken hoping to identify different levels of thinking by students working on challenging tasks. With all classroom-based research, uncertainty in data is likely, which is especially the case in this small study limited by conducting research in one classroom in one school in a short timeframe. For these reasons, any claims cannot be generalised. However, the findings suggest that students demonstrate many types of thinking when working on challenging tasks. This study suggests that it is possible to assess the depth of thinking that students engage in at that time. It also appears that teachers can support students in refining their thinking and their ability to record strategies and reasoning.

There are implications for the classroom. These include the potential of teacher-developed rubrics that support observation and timely feedback. Feedback may move students from their current level of conceptual understanding, broaden the range of strategies they are comfortable in using and encourage clear explanations of thinking. Teachers could develop a progression of strategies towards a conceptual understanding of multiplication and division that incorporate what they have observed in their students.

This small study could be used to inform a larger study around using evidence-informed practice and formative assessments to improve teaching and learning for a range of different mathematical concepts. Further research could include refining the descriptive sections of the rubric and developing challenging tasks for use in other areas of mathematics. Consideration would need to be given to other elements involved in the successful implementation of challenging tasks, including encouraging persistence, fostering the skills of listening to others, and teachers reflecting on their and students' experiences. Loong et al (2018) published a study after this research was completed and produced an Assessing Mathematical Reasoning rubric. This rubric helped teachers to support their understanding of how to develop reasoning in students and report their progress. Further research is needed to ensure that rubrics such as the one in this study and in Loong et al. (2018) are pragmatic, time efficient and provide appropriate information.

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Appendices

Appendix A

Rubric for levels of thinking used in exploring challenging mathematical tasks

Level of Thinking	**Problem Solving**	**Reasoning**	**Representation**	**Connection**
Level 1 Recall and reproduction	Did not understand the task. What did the student appear to interpret the task as entailing?	Mathematical thinking is incorrect.	Used no mathematical language and/or notation. Diagrams did not relate to the task.	Did not make any connections to the task or the numbers in the task.
Level 2 Basic application of skills and concepts	Understood part of the task. Needed help to understand the entire task. Their strategy worked for part of the task.	Some mathematical thinking or explanation is correct. Needed help to explain the task.	Used some mathematical language and/or notation. Some diagrams were used to represent the task.	Tried to make some connections to previous learning related to the task.
Level 3 Strategic thinking	Understood the task and the strategy they used worked.	Mathematical thinking and explanation is correct. Some thinking was systematic.	Used clear mathematical language and/or notation throughout their work. Diagrams clearly related to the task.	Made some mathematical connections to previous learning related to the task and recorded it in some way.
Level 4 Extended thinking	Understood the task. Used an efficient strategy. Extension activities, if available, were completed.	Gave a detailed and accurate explanation of the strategy/method they used to solve the task. All mathematical thinking was correct and systematic. Extension activities, if available, were completed.	Used a range of specific math language and/or notation throughout their work. Diagrams related directly to the task and used to explain student thinking. Extension activities were completed.	Recorded mathematical connections to mathematical big ideas and strategies previously used. Extension activities were completed.

Appendix B

- Enabling prompt examples.
- Reducing the starting numbers for the tasks – ‘The Doughnut Tree’ task; starting with 32 or 64 students in ‘The Big (not so) Friendly Giant task.
- Providing concrete materials.
- Altering the task depiction for students – breaking The Doughnut Tree task into one day at a time.
- Reducing the number of steps – just presenting students with Task 1 in each case.
- Altering task presentation expectations – supporting students with recording their ideas.

Appendix C

Whole class and student discussion notes by preservice teacher from launch phase.

Teacher: What do you think the word ‘challenging’ means?

Student 1: I don’t know YET, how to do this but I can learn.

Student 19: Try your best and keep working at it.

Student 5: When you go to university you need to make sure you can teach challenging tasks.

Student 8: You need to have persistence.

Appendix D

Discussion notes made by the PSST, from explore phase.

Student 21	Understood how to double numbers. Could demonstrate the number of days that are in the 2 weeks. Struggled in showing the strategies they could use to show how to double larger numbers.
Student 5	Finished the task quickly but the preservice teacher had seen that the student did not read the question properly. Understood that 2 weeks has 14 days. Changed their idea of thinking when the preservice teacher read through the questions with them. The student had assumed they couldn’t use the rule of doubling. When Student 5 read through the question, they changed their answers and began to use the doubling strategy.
Student 26	Doubled every number on the calendar. Knew that in 2 weeks there are 14 days. Drew doughnuts on the page to find out each answer when doubling. Stated, ‘When numbers get large I am going to use a new strategy’. The strategy the student chose was using number lines. They counted up by 10s and then added the remainder.

Appendix E

What else belongs?

Task 1	Task 2
What is the same about these numbers? Could these numbers belong together? What are the reasons these numbers might be together in a group? Complete the sentence starter: These numbers belong together because …	Complete these sentence starters: Other numbers that belong in this group are … These numbers also belong in the group because …

EXTENSION: Make your own lists of 3 numbers you could give to someone else for them to decide on reasons for the group.