Promoting the acquisition of higher-order skills and understandings in primary and secondary mathematics

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Abstract

What do we mean by higher-order skills? How do students develop higher-order skills, and utilise abstract ideas or concepts? How can we promote the acquisition of higher-order understandings in a classroom situation? This session considers these questions and the reasons for the difficulties and challenges teachers face in addressing the need to promote higher-order understandings in their students. The research reported draws on data from three large-scale longitudinal studies carried out with primary and secondary teachers. The approaches are consistent with recent research findings on cognition and brain functioning, and provide insight into how such skills are developed in students. Participants will consider practical ways to create conditions that increase the likelihood of higher-order skills and understandings in their students.

Introduction

There is little evidence of systematic use of cognitive-based research to influence wide-scale curriculum developments, or their associated assessment and instruction practices (Pegg & Panizzon, 2001). Significantly, and central to this paper, if assessment and teaching practices are to improve, then such practices must rest on theoretical bases for learning which provide useable information to teachers to guide their thinking and subsequent teaching actions (Pellegrino, Chudowsky, & Glaser, 2001).

Further, any theoretical position adopted must be empirically based and not simply rely on 'logic' for its rationale. The theory must offer teachers the opportunity to achieve the synchronisation of the three arms of curriculum – assessment, pedagogy, and syllabus content – thus achieving 'constructive alignment' (Biggs, 1996). It is the position of the author that the SOLO (Structure of the Observed Learned Outcome) model (Biggs & Collis, 1982; 1991; Pegg, 2003) meets these requirements and provides a theoretical underpinning for assessment and instruction decisions taken by teachers.

The ideas reported here draw on data from three large-scale longitudinal studies, involving the SOLO framework, with primary and secondary teachers in NSW. This paper draws from these studies ideas associated with the development of higher-order skills and understandings. The use of SOLO emphasises the integral role assessment practices play as part of normal classroom activity with the information obtained being used to inform, monitor and promote student learning (Black & Wiliam, 1998).

The findings of these studies illustrated dramatically the value such a framework plays when groups of teachers interpreted student responses to assessment tasks and plan how responsive instruction might proceed. Without a framework such as SOLO, teachers could offer little guidance on how they might decide consistently and across a range of activities whether assessment items were appropriate, whether student responses to assessment items were adequate, what skills and understandings students possessed, and where instruction might be directed most profitably in the future.

In this paper we consider: What is meant by higher-order skills? How will students acquire higher-order skills and utilise abstract ideas or concepts? In what ways can we promote the acquisition of higher-order skills and understandings in a classroom?

Higher-order skills and understandings

What do we mean by higher-order skills and understandings? Probably the best-known description is offered by Bloom's Taxonomy, named after the leader of the group of academics in 1956 that released the *Taxonomy of Educational Objectives*. There are six categories to Bloom's Taxonomy. These are: knowledge, comprehension, application, synthesis, analysis and evaluation. Knowledge and comprehension are seen as important lower-level skills and are concerned with remembering information and basic understanding. Higher-order skills involve application (using knowledge), analysis, synthesis and evaluation.

While Bloom's Taxonomy has come under increasing criticism leading to review (Anderson et al., 2001), the basic ideas still offer help to teachers, in advance of testing, to identify assessment items that target different categories of quality. The issue here is that the category of a particular question does not usually provide insight into the level of a student's response.

SOLO adopts a different position, namely, that 'there are "natural" stages in the growth of learning any complex material or skill' (Biggs & Collis, 1982, p. 15). The model seeks to describe this growth sequence through a series of *modes* of understanding and levels of performance within these modes. SOLO levels provide teachers with a convenient way to label portions of the continuum for practical purposes.

SOLO model

The relevance of SOLO to higher-order functioning is that it is an empirically verifiable assessment framework designed for use in classrooms. Over the past 30 years, SOLO has built a substantial empirical base involving numerous research studies resulting in many hundreds of published articles. SOLO is a model for categorising the responses of students in terms of structural characteristics.

The focus of the SOLO categorisation is on cognitive processes rather than the end products alone. The task of the teacher is to analyse the pattern of ideas presented by the student. SOLO facilitates the successful completion of this task by providing a balance between structural complexity and content/context. In SOLO, development is dependent upon the nature or abstractness of the task (referred to as the mode) and a person's ability to handle, with increased sophistication, relevant cues (referred to as the level of response).

SOLO comprises five *modes of functioning* referred to as sensori-motor, iconic, concrete symbolic, formal and post formal. Learning can occur in one of these modes or be multi-modal. Within each mode are series of three levels of response. A *unistructural* response is one that includes only one relevant piece of information from the stimulus; a *multistructural* response is one that includes several relevant independent pieces of information from the stimulus; and a *relational* response is one that integrates all relevant pieces of information from the stimulus. These three levels comprise a U-M-R cycle of development.

Having achieved a relational level response in one cycle, students move to the next level that represents a new unistructural level in a new cycle. This enhanced unistructural response represents (i) a consolidation of the previous relational response into a single more succinct form within the same mode, or (ii) a new unistructural response that not only includes all relevant pieces of information, but also extends the response to integrate relevant pieces of information not in the stimulus that are typical of the next mode of understanding.

The strength of the SOLO model is the linking of the hierarchical nature of cognitive development through the modes and the cyclical nature of learning through the levels. Each level provides building blocks for the next higher level. SOLO also provides teachers with a common and shared language that enables them to describe in a meaningful way their observations of student performance. This is particularly important when teachers try to articulate differences between lower-order and higher-order skills and understandings.

SOLO and higher-order functioning

The most common modes for instruction for primary and secondary mathematics are the concrete symbolic mode (becoming available on average about 5–6 years of age) and the formal mode (becoming available around 15–16 years of age). In SOLO the levels are ordered within a mode, with students entering the field picking up single aspects, then multiple but independent aspects, and finally integrating these separate aspects into a cohesive whole.

It is the answers coded at the unistructural and multistructural levels that are seen as lower-order responses. Here the students recall single or multiple ideas, know basic facts, and are able to undertake routine tasks by applying standard algorithms.

Higher-order skills commence at the relational level. This arises through the ability to integrate information and make personal connections resulting in using this knowledge in related but new areas. Here students are able to: demonstrate some flexibility in their work; undertake problems without relying on step-by-step learnt algorithms; see novel connections not previously taught; have an overview of the concept under consideration and how different aspects of the concept are linked; show insight – able to undertake 'new' questions; and provide reasonable evidence of understanding.

The relational level response is a precursor to more abstract thinking that occurs in the subsequent mode (the formal mode) where students are able to work with relationships between concepts as their thought processes become more abstract and they move away from the need for concrete referents. They are able to formulate their own hypotheses, develop their own models, work in terms of general principles, and construct their own mathematical arguments.

Ideas about cognitive architecture

What determines the SOLO levels for particular students? The answer seems to encompass six main ideas. These are: general cognitive abilities of the student; familiarity of the content; presentation of the task; degree of interest or motivation of the student; amount of relevant information that can be retained simultaneously for this task; and the amount of information processing required for a solution.

These last two points are particularly important to this discussion as they lead to the notion of *working memory*. Working memory is a theoretical construct and is usually defined as the ability to hold information in the mind while transforming or manipulating it. Working memory is used to organise, contrast, compare, or work on information. Working memory is limited in capacity and duration. As we become more expert in a task, our working memory capacity does not increase but it does become more efficient.

There is some conjecture about the relationship between working memory and both short-term and long-term memory. The current consensus is that working memory and short-term memory are distinct. Short-term memory is associated with information that is held for short periods of time and reproduced in an unaltered fashion. Long-term memory is where permanent knowledge is stored for long periods of time. Individuals access and work on this stored knowledge through their working memory.

Implications for learning I

- Human intelligence comes from stored knowledge in long-term memory, not long chains of reasoning in working memory.
- Skilled performance consists of building chains of increasingly complex schemas in long-term memory by combining elements consisting of low-level schemas into high-level schemas.
- A schema can hold a huge amount of information as a simple unit in working memory.
- Higher-order processing occurs when there is 'sufficient space' in working memory so that appropriate schemas can be accessed from long-term memory and worked upon.

Implication for learning II

- Improved automaticity in fundamental/basic skills, such as calculating, at lower levels frees up working memory resources for processing higher-order skills and understandings.
- Deliberate practice at the unistructural level reduces the demands of working memory on these concepts.
- If at the unistructural level, working memory demands are reduced, the growth of multistructural responses is facilitated.
- Freeing up of resources at lower levels allows students to focus on inherently attention-demanding higher-order cognitive activities.

Implications for learning III

- At the unistructural and multistructural levels relevant information can be 'taught' in the traditional sense.
- At the relational level, 'teaching' in a traditional sense is problematic as students need to develop their own connections – their own way.
- Language development is important in developing students' understanding and reducing working memory demands at the multistructural level – establishing a strong basis for relational responses.
- Students can respond by rote at relational levels without understanding and hence give the impression of having attained higher-order skills.

Implications for teaching

Once students can respond consistently at the multistructural level, with appropriate language skills, teachers should focus on creating an environment to promote SOLO relational responses. Such an approach encourages students to integrate their understanding of individual ideas and see connections and elaborations not previously met. Attempting non-routine problems is one important way in achieving high-order skills and understandings as, in general, these questions require at least relational responses. Generally, with non-routine questions, there are no prescribed algorithmic approaches.

Examples of how to generate such environments include providing students with:

- the answer to a problem and having them generate questions, i.e., reversibility
- more information than the question/problem requires
- less information than the question/problem requires.

Conclusion

Higher-order skills and understandings are more difficult to learn and to teach, as they require more cognitive processing and different forms of instruction. Such skills and understandings are prized as they allow knowledge to be owned by the individual and, hence, applied in novel ways to different situations. Teachers should orchestrate, at the appropriate times, environments for higher-order mathematical thinking activities to take place on the syllabus content being covered in class.

For the successful development of higher-order skills and understandings, activities of instruction and assessment need to be closely intertwined. In particular, formal testing and informal formative assessments need to inform teaching. Considering assessments this way will help teachers understand where students are in their learning journey, and better facilitate the focus of instruction to meet the actual needs of students.

Important in this movement from lower-order to higher-order skills and understandings is the use of an evidence-based cognitive framework. This paper advocates the SOLO model as one suitable framework. With such a model, teachers have at their disposal signposts along a continuum of cognitive development. One obvious consequence is that such a framework helps explain when it is most appropriate to address higher-order skills and understandings, and when to consider different instructional strategies as students move through levels acquiring new knowledge.

An implication of the SOLO hierarchy is that higher-order skills and understandings in the mathematics classroom are built upon the acquisition of lower-order skills and understandings. They have a symbiotic association in which: (i) the relational level represents the start of higher-order functioning; and (ii) the unistructural level represents higher-order functioning for an earlier growth cycle and at the same time the beginning of lower-order functioning in the current cycle.

Finally, working from a developmental cognitive perspective, such as the SOLO model, *exposes as fanciful and counter productive* 'commonsense' expectations of teachers: 'that almost all the time their students should be engaged in higher-order thinking'.

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