Promoting the acquisition of higher-order skills and understandings in primary and secondary mathematics

John Pegg

The National Centre of Science, Information and Communication Technology, and Mathematics Education for Rural and Regional Australia
Background to Paper

• Little evidence of cognitive-based research influencing wide-scale curriculum developments (includes brain studies)

• There needs to be constructive alignment (Biggs) assessment, pedagogy and syllabus content

• Evidence to the Ideas here come from research on three large-scale studies involving primary and secondary teachers

• SOLO Model offers the best approach/framework thus far to help teachers do a better job
Plan for Talk

What is meant by higher-order skills? **Set the scene**

How students acquire higher-order skills, and utilize abstract ideas or concepts? **Consider the brain and learning**

Ways we can promote the acquisition of higher-order skills and understandings in a classroom. **Work through some practical teaching ideas for classroom use**
Higher order skills, what are they?

Best-known description is offered by Bloom’s Taxonomy, named after the leader of the group of academics in 1956.

There are six categories to Bloom’s Taxonomy. These are: knowledge, comprehension, application, synthesis, analysis and evaluation.

Knowledge and comprehension are seen as important lower-level skills concerned with remembering information and basic understanding.

Higher-order skills involve application (using knowledge), analysis, synthesis and evaluation.
Higher order skills, what are they?

Bloom’s Taxonomy has come under increasing criticism leading to review (Anderson et al., 2001)

Basic ideas still offer help to teachers, in advance of testing, to identify assessment items that target different categories of quality.

The issue here is that the category of a particular question does not usually provide insight into the level of understanding of a student’s response.
Some issues about cognitive processing

OR

Implications of SOLO for teaching
“We believe that there are ‘natural’ stages in the growth of learning any complex material or skill and that in certain important aspects these stages are similar to, but not identical with, the developmental stages in thinking described by Piaget and his co-workers.”

(Biggs & Collis, 1982, p. 15)

• Systematic and objective qualitative assessments of student learning are desirable and necessary.

• Qualitative assessments may be made in terms of the ‘structural complexity’ of the student response or outcome.
A system to classify the QUALITY of a response.
Background to SOLO

Assessment is concerned with two big ideas:

- How much has been learned
- How well has it been learned

To describe these ideas we use:

- quantitative (how much)
- qualitative (how well)

Both forms of assessment are important. Traditionally ‘how much’ determines the grade because it is easier.
## Descriptions of Modes in the SOLO Model

<table>
<thead>
<tr>
<th>Mode</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensorimotor: (soon after birth)</td>
<td>A person reacts to the physical environment. For the very young child it is the mode in which motor skill are acquired. These play an important part in later life as skills associated with various sports evolve.</td>
</tr>
<tr>
<td>Ikonic: (from 2 years)</td>
<td>A person internalises actions in the form of images. It is in this mode that the young child develops words and images which can stand for objects and events. For the adult this mode of functioning assists in the appreciation of art and music and leads to a form of knowledge referred to as intuitive.</td>
</tr>
</tbody>
</table>
Concrete
Symbolic:  
(from 6 or 7 years)

A person thinks through use of a symbol system such as written language and number systems. This is the most common mode addressed in learning in the upper primary and secondary school.

Formal: 
(from 15 or 16 years)

A person considers more abstract concepts. This can be described as working in terms of ‘principles’ and ‘theories’. Students are no longer restricted to a concrete referent. In its more advanced form it involves the development of disciplines.
Descriptions of Modes in the SOLO Model

Post Formal: (possibly at around 22 years)

A person is able to question or challenge the fundamental structure of theories or disciplines.

Overall, there are five modes
Modes, Learning Cycles and Forms of Knowledge

- **Mode**
  - Sensori-Motor
  - Ikonic
  - Concrete Symbolic
  - Formal
  - Post Formal
- **Forms of Knowledge**
  - Tacit
  - Intuitive
  - Declarative
  - Theoretical

- **Course of Optimal Development**
- **Lower Order Learning**
- **Higher Order Learning**

- **Uni-modal development**

Age (years not to scale)
Description of Levels in the SOLO Model

Unistructural:
The student focuses on the domain/problem, but uses only one piece of relevant data and so may be inconsistent.

Multistructural:
Two or more pieces of data are used without any relationships perceived between them. No integration occurs. Some inconsistency may be apparent.

Relational:
All data are now available, with each piece woven into an overall mosaic of relationships. The whole has become a coherent structure. No inconsistency within the known system.
What is a cycle?

A cycle is a sequence of three levels.

These three levels are: **unistructural**, **multistructural** and **relational** levels.

These levels occur within the ‘targeted’ mode.
Two Cycles Within a Mode

Concrete
Symbolic
Mode

U₁ → M₁ → R₁ → M₂ → U₂

U₁ → M₁ → R₁ → M₂ → U₂
A two-cycle diagram
Body of knowledge to be ‘taught’
A Fundamental Learning Cycle (Pegg & Tall)

Unistructural responses A or B or C or D
Multistructural responses A and B and C and D (or a subset)
Relational responses A B C D or A C B D or B D A C or...

- This three-level UMR cycle reoccurs over and over again.
- The key point is that the relational level represents the higher-order response of any particular cycle.
How are these levels determined for a particular task?

1. General cognitive abilities of the student
2. Familiarity of the content
3. Presentation of the task
4. Degree of interest or motivation of the student
5. Amount of relevant information that can be retained simultaneously for this task
6. Amount of information processing required for solution

5&6 Lead to the notion of *Working Memory (WM)*
Recent ideas about Cognitive Architecture

Long term memory (LTM) is what people use to make sense of, and give meaning to, what they are doing now. It is a repository of permanent knowledge.

LTM can be considered unlimited. We are not directly aware of LTM. Its contents and functioning is filtered through working memory (WM).

Short term memory (STM) is typically used to describe instructions in which small amounts of memory are held passively for a short period of time and then reproduced in a sequential and untransformed fashion.
Working Memory (WM)

WM is usually defined as the ability to hold information in the mind while transforming it, or other information.

WM is used to organise, contrast, compare, or work on information.

WM is limited in capacity and duration. As we become more expert in a task, our WM does not increase but becomes more efficient.

A person can only process 2 or 3 items of information simultaneously as opposed to merely holding about 7 bits of information at a time.
Parts of WM

WM is not a monolithic structure. It has at least two mod-specific components and a central executive controlling system (Baddeley).

1. A visual - spatial sketchpad - plays a key role in the generation and manipulation of mental images - mainly in the right hemisphere.

2. A phonological loop - temporary storage of verbal information and subvocal articulation - mainly in the left hemisphere.

3. Central executive system coordinates these two systems, focusing and switching attention and activating representations within LTM - mainly in the pre-frontal cortex.
Implications for learning 1

• Human intelligence comes from stored knowledge, not long chains of reasoning in WM

• Skilled performance consists of building increasing numbers of increasingly complex schemas by combining elements consisting of low level schemas into high level schemas

• A schema can hold a huge amount of information as a simple unit in working memory

• Higher-order processing occurs when there is ‘sufficient space’ in working memory so that appropriate schemas can be accessed from long-term memory and worked upon.
Implications for learning 2

• Improved automaticity in fundamental/basic skills, such as calculating, at lower levels frees up working-memory resources for processing higher-order skills and understandings.

• Deliberate practice at the unistructural level reduces the demands of working memory on these concepts.

• If at the unistructural level, working memory demands are reduced, the growth of multistructural responses is facilitated.

• Freeing up of resources at lower levels allows students to focus on inherently attention-demanding higher-order cognitive activities.
Automaticity in Action

**AUTOMATICITY** is the ability to complete everyday tasks effortlessly with low interference of other simultaneous activities and without conscious thought to step-by-step process.

When the brain recognizes familiar tasks it processes the information and applies the correct rules to the procedure in order to reduce the demand on working memory and allow for higher-order processing of information.
Figure 1 shows how a scan of the brain might look before automaticity of a skill is acquired. The working memory is actively operating to process and categorize information.

Figure 2 shows what the scan of a brain might look like after automaticity of a skill is acquired. There is a drastic reduction of activity in brain.
Implications for Teaching 1

We need to have different teaching approaches for different stages in the learning process

- At the unistructural and multistructural levels relevant information can be ‘taught’ in the traditional sense.

- At the relational level, ‘teaching’ in a traditional sense is problematic as students need to develop their own connections – their own way.
Theoretical Orientation: SOLO model

Teacher actions:

Explicit teaching required to come to know the individual elements needed, and to practice and consolidate

Creating an environment for students to make the links

Fundamental learning cycle:

- **U (unistructural)** focuses on one aspect
- **M (multistructural)** focuses on several aspects independently
- **R (relational)** integrates the several aspects
- **new U (unistructural)** consolidates the previous R into a new element
Implications for teaching 2

• Language development is important in developing students’ understanding and reducing working-memory demands at the multistructural level – establishing a strong basis for relational responses.

• Students can respond by rote at relational levels without understanding and hence give the impression of having attained higher-order skills.
Implications for teaching 3

Once students can respond consistently at the multistructural level, with appropriate language skills, teachers should focus on creating an environment to promote SOLO relational responses.

Such an approach encourages students to integrate their understanding of individual ideas and see connections and elaborations not previously met.

Attempting non-routine problems is one important way in achieving high-order skills and understandings as, in general, these questions require at least relational responses. (Generally, with non-routine questions there are no prescribed algorithmic approaches.)
What is a big teaching idea?

Reversibility

- Involves giving the students an answer and asking: What is the question?
- In general, these are relational questions
- Encourages students to integrate their understanding of individual ideas
- Generally, there are not prescribed algorithmic approaches for these types of questions
Big Idea for teaching

Mathematics

A favourite of mine

Find the area of the following triangles
What is the big teaching idea?

Providing students with questions with Too much or Too little information.

But to focus students attention on what is needed to solve the question.

This requires an overview of the question - a relational understanding.
Conclusion

Higher-order skills and understandings are more difficult to learn and to teach, as they require more cognitive processing and different forms of instruction.

For the successful development of higher-order skills and understandings, activities of instruction and assessment need to be closely intertwined.

Important in this movement from lower-order to higher-order skills and understandings is the use of an evidence-based cognitive framework.
An implication of the SOLO hierarchy is that higher-order skills and understandings in the mathematics classroom are built upon the acquisition of lower-order skills and understandings.

Working from a developmental cognitive perspective, such as the SOLO model, exposes as fanciful and counter productive ‘commonsense’ expectations on teachers: “that almost all the time their students should be engaged in higher-order thinking.”
Some Properties of Levels

- People reasoning on different levels speak a different language and in general cannot understand one another. This can occur between students within the same classroom.

- Very serious communication problems exist between students on one level and their fellow students, teachers, text-books, and exercises on another level.

- Students need to confront a personal ‘crisis of thinking’ in moving from one level to the next. They cannot be forced to think at a higher level.

- Certain teaching strategies can inhibit such growth and place boundaries on students’ potential.
• A student cannot attain a higher level without first passing through the lower level(s).

• However, students can simulate higher levels by learning rules or definitions by rote or by applying routine algorithms that they do not understand.

• ‘Level reduction’ occurs when structures at a higher level are re-interpreted at a lower level. This procedure, when it is teacher directed, can be counter productive as it can remove the stimulus for students to attain a higher level.

• Level reduction needs to be student owned.