Using mental representations of space when words are unavailable: Studies of enumeration and arithmetic in Indigenous Australia

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Abstract

Here we describe the nature and use of spatial strategies in a standard non-verbal addition task in two groups of children, comparing children who speak only languages in which counting words are not available with children who were raised speaking English. We tested speakers of Warlpiri and Anindilyakwa aged between 4 and 7 years old at two remote sites in the Northern Territory of Australia. These children used spatial strategies extensively, and were significantly more accurate when they did so. English-speaking children used spatial strategies very infrequently, but relied an enumeration strategy supported by counting words to do the addition task. The main spatial strategy exploited the known visual memory strengths of Indigenous Australians, and involved matching the spatial pattern of the augend set and the addend. These findings suggest that counting words, far from being necessary for exact arithmetic, offer one strategy among others. They also suggest that spatial models for number do not need to be one-dimensional vectors, as in a mental number line, but can be at least two-dimensional.

Introduction

Indigenous Amazonians, whose languages lack our kind of 'count-list', appear unable to accurately carry out tasks that require 'the capacity to represent integers' (Gordon, 2004; Pica, Lemer, Izard, & Dehaene, 2004). The Amazonian researchers, therefore, claim that 'Language would play an essential role in linking up the various nonverbal representations to create a concept of large exact number' (Pica et al., p. 499) and conclude 'Our results thus support the hypothesis that language plays a special role in the emergence of exact arithmetic during child development' (Pica et al., p. 503). This is a Whorfian position: concepts of exact number are impossible without counting words. That is, one cannot possess the concept of exactly fiveness, without having a word corresponding to five.

This view is not universal. Gelman and Gallistel (1978) argue that the child's development of verbal counting is a process of mapping a stably ordered sequence of counting words (CW) onto an ordered sequence of mental marks for numerosities they call 'numerons'. This system is shared with non-verbal species such as crows and rats, and is implemented in an 'accumulator' system that accumulates a fixed amount of neural energy or activity for each item enumerated. Each numeron corresponds to a level of the accumulator.

One can think of the mental number line (MNL) as being a scale that is calibrated against the accumulator. Similarly, one can think of the count list as being lined up against points or regions on the MNL. Spatial metaphors of abstract concepts and relations are extremely widespread in human cognition: emotions are described as high or low, personal relationships can be close or distant, most people go forward into the future, backward into the past, etc. It is not therefore surprising that cardinal numbers, which are abstract properties of sets, should attract spatial models. The unconscious spatial representation of numbers, revealed in number bisection tasks, is usually thought of as one-dimensional vectors – a line with a single direction. However, where individuals have automatic and conscious representations of number – Galton's ‘number forms’ (Galton, 1880) – these are indeed lines, but more complex, in two or even three dimensions (Seron, Pesenti, Noël, Deloche, & Cornet, 1992; Tang, Ward, & Butterworth, 2008).

Here we ask the question: what will individuals do when they do not have counting words in tasks that require exact calculation? The Whorfian position would entail that exact calculation is impossible. On the other hand, the position espoused by Locke (Locke, 1690/1961) and Whitehead (Whitehead, 1948), and subsequently by Gelman and Butterworth (2005), is that ‘Distinct names conduce to our well reckoning’ because, as Whitehead notes, ‘By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and in effect increases the mental power of the race’ (Whitehead, 1948).

Are CWs the only ‘good notation’? Here we examine the ability of Indigenous Australian children of 4 to 7 years to carry out simple non-verbal addition problems. These children lived in remote sites in the Northern Territory, and were monolingual in one of two Australian languages, Warlpiri or Anindilyakwa. These languages have very limited number vocabularies. Although these languages contain quantifiers such as *few*, *many*, *a lot*, *several*, etc., these are not relevant number words, since they do refer to exact numbers, and the theoretical claim is about exact numbers. Our comparison group was a school in Melbourne.

We have already shown that these children perform accurately as English-speaking children on tasks that required remembering the number of objects in an array and on matching the number of sounds with a number of objects (Butterworth & Reeve, 2008; Butterworth, Reeve, Reynolds, & Lloyd, 2008). Here we focus on a non-verbal exact addition task. Addition is typically acquired in stages using counting procedures. Where two numbers or two disjoint sets, say 3 and 5, are to be added together, in the earliest stage the learner counts all members of the union of the two sets – that is, will count 1, 2, 3, and continue 4, 5, 6, 7, 8, keeping the number of the second set in mind. In a later stage, the learner will ‘count-on’ from the number of the first set, starting with 3 and counting just 4, 5, 6, 7, 8. At a still later stage, the child will count on from the larger of the two numbers, now starting at 5, and counting just 6, 7, 8. (Butterworth, 2005). It is probably at this stage that addition facts are laid down in long-term memory (Butterworth, Girelli, Zorzi, & Jonckheere, 2001). If the learner does not have access to these strategies, because his or her language lacks the CW, what will they do? (Note: Many learners during these stages use their fingers – a handy set – to help them count, especially when the addition involves numbers rather than sets of objects. That is, they will represent the 3 by raising three fingers, and then count on using the five fingers of the other hand. Now, despite the fact that many cultures with no specialised number words use body-parts and body-part names to count, this is not what happens in Australia. Although gestural communications are very widespread there (Kendon, 1988), there is no record of body-part counting or of showing numbers using body-parts. This seems to be a conventional form of communication that is lacking in Australia. Indeed, none of our Northern Territory children used their fingers to help them with these tasks.

Method

We tested 32 children aged 4 to 7 years: 13 Warlpiri-speaking children, 10 Anindilyakwa-speaking children, and 9 English-speaking children from Melbourne. Approximately half the Northern Territory children were 4 to 5 years old and half were 6 to 7 years old.

In Willowra and Angurugu, bilingual Indigenous assistants were trained by an interviewer to administer the tasks, and all instructions were given by a native speaker of Warlpiri or Anindilyakwa. To acquaint helpers with research practices and to familiarise children with test materials (e.g., counters), familiarisation sessions were conducted. Children played matching and sharing games using test materials (counters and mats). For the matching games, the interviewer put several counters on her mat, and children were asked to make their mat the same. Children had little difficulty copying the number and location of counters on the interviewer’s mat.

In the basic memory task, identical 24-cm × 35-cm mats and bowls containing 25 counters were placed in front of a child and the interviewer. The interviewer sat beside the child, as recommended in Kearins (1981), rather than opposite as is typical in testing European children. The interviewer took counters from her bowl and placed them on her mat, one at a time, in pre-assigned locations. Four seconds after the last item was placed on the mat, all items were covered with a cloth and children were asked by the Indigenous assistant to ‘make your mat like hers’. Following three practice trials in which the interviewer and an Indigenous assistant modelled recall using one and two counters, children completed 14 memory trials comprising two, three, four, five, six, eight, or nine randomly placed counters. In modelling recall, counters were placed on the mat without reference to their initial location. Number and locations of children’s counter recall were recorded. In earlier analyses we found that Indigenous children tended to use spatial strategies to reconstruct the numerosities of random memory arrays (Butterworth & Reeve, 2008). Of interest is whether they would use similar strategies in the non-verbal addition task.

The same materials (mats and counters) were used in the non-verbal addition task. The interviewer placed one counter on her mat and, after 4 seconds, covered her mat. Next, the interviewer placed another counter beside her mat and, while the child watched, slid the additional counter under the cover and onto her mat. Children were asked by the Indigenous assistant to 'make your mat like hers'. Nine trials comprising 2 + 1, 3 + 1, 4 + 1, 1 + 2, 1 + 3, 1 + 4, 3 + 3, 4 + 2, and 5 + 3 were used. Children's answers were recorded. We were particularly interested in the ways in which computed answers to the non-verbal addition problems were approached, and in whether Indigenous children would use spatial strategies in computing answers.

Results

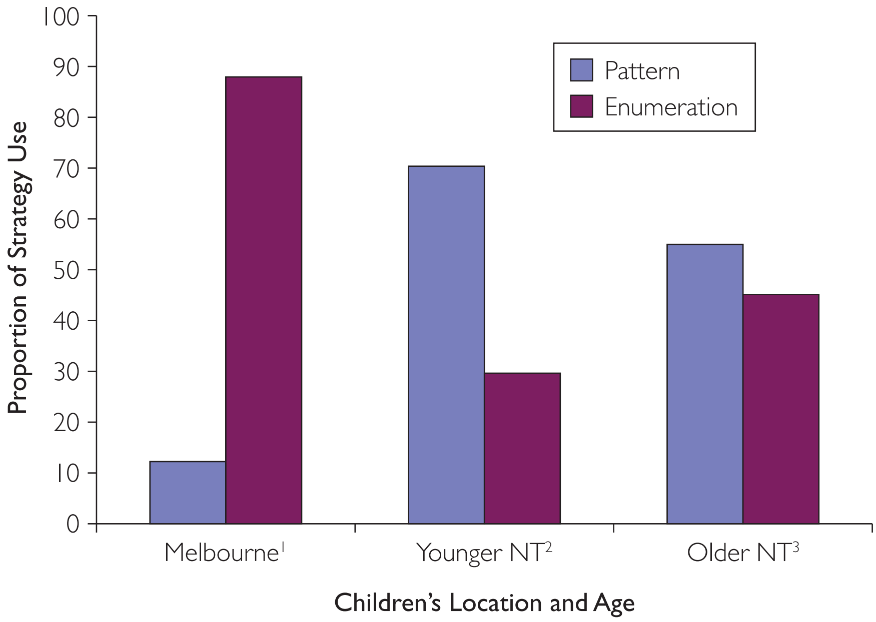
The patterns of findings are reasonably clear. Compared to their Melbourne peers, the younger Northern Territory children solved marginally more non-verbal addition problems correctly (means = 2.3 and 3.2 problems correct respectively, F (1, 20) = 3.27, $p < .09$). Further, the older Northern Territory children solved more problems correctly than the younger Northern Territory children (means = 3.2 and 4.5 problems respectively, F (1, 23) = 10.15, $p < .01$).

Strategies

Of interest are differences in the strategies used to solve the non-verbal addition problems by the different groups of children (Melbourne vs Northern Territory, and younger vs older Northern Territory children) and whether these differences, if they exist, affect problem-solving success. The strategy used to solve each problem was classified as either an enumeration or a pattern strategy. For a problem-solving attempt to be classified an enumeration strategy, the tokens used to convey answers were placed by the child on his or her mat in a random or linear arrangement (often with audible enumeration). For a problem-solving attempt to be classified a pattern strategy, a child appeared to concatenate the two patterns (the original token pattern, and the pattern of added tokens). The pattern strategy reflects an attempted reproduction of the spatial layout of the initial and added arrays. In this case, no audible enumeration accompanied token placement. These two strategies appear to reflect two meaningfully different computation processes.

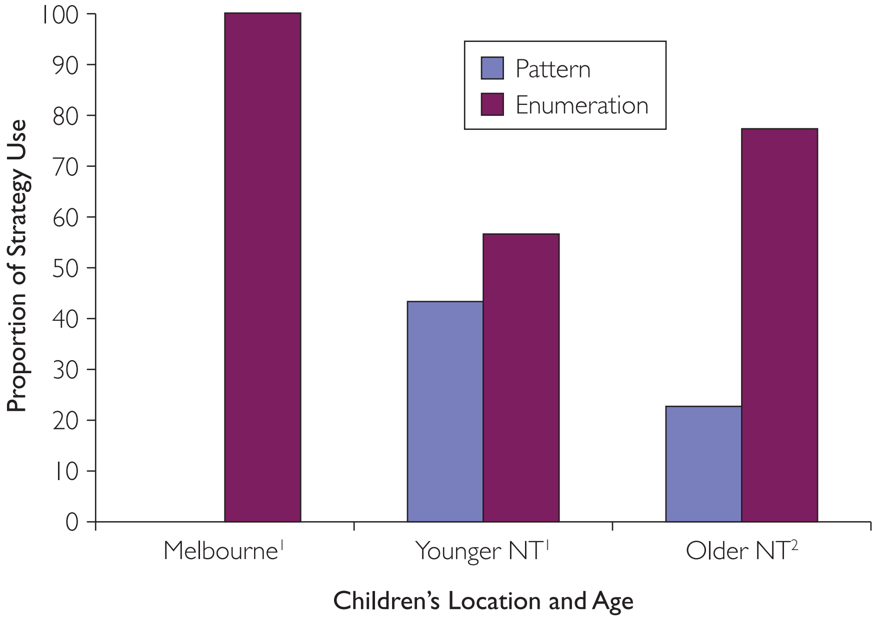
When problems were solved correctly, Melbourne children used enumeration strategies more often than their young Northern Territory peers, who used pattern strategies more often, χ^2 (1, N = 56) = 18.08, $p < .001$. Similarly, when correct, older Northern Territory children used an enumeration strategy more often than younger NT children, χ^2 (1, N = 57) = 4.30, $p < .05$. For incorrectly solved problems, the results were reversed for Melbourne and young Northern Territory children: young Northern Territory children tended to err when they used an enumeration strategy, χ^2 (1, N = 62) = 14.91, $p < .001$.

Figures 1 and 2 show strategy use for correct and incorrect answers as a function of age and test location. Figure 1 shows that Melbourne children are more likely to obtain the correct answer if they used an enumeration strategy ($p < .01$), and that this effect is reversed for the younger Northern Territory children ($p < .05$). However, older Northern Territory children's correct non-verbal addition problem-solving ability does not seem to depend on strategy use. However, Figure 2 shows that older Northern Territory



[1] $p < .01$, [2] $p < .05$, 3 n.s.

Figure 1: Proportion of strategy use for correct nonverbal addition responses as a function of children's location and age



[1] n.s, [2] $p < .05$

Figure 2: Proportion of strategy use for incorrect nonverbal addition responses as a function of children's location and age

children are more likely to err if they used an enumeration strategy ($p < .05$).

Discussion

It is clear that English-speaking children in Melbourne almost never use the pattern strategy, but perform the task using an enumeration strategy. By contrast, Northern Territory children matched in age with the English-speakers, use pattern strategies nearly twice often as enumeration. What is of particular interest is the fact that the pattern strategy is more effective for them, and that attempting to enumerate leads to a preponderance of errors. Indeed, even for the English-speakers, the only four documented uses of pattern were all correct. The older Northern Territory children have begun to use the pattern strategy more often, now making up about half of all strategies used. However, the majority of their correct responses (30 vs 24) and the minority of their incorrect responses (5 vs 13) used the pattern strategy.

These results suggest that a pattern-matching strategy is an effective spatial heuristic when CWs to support enumeration are not available. Notice that the patterns used here are two-dimensional, suggesting that a one-dimensional oriented number line is not the only way for children to represent numbers. One might ask why pattern matching is the preferred strategy for the Northern Territory children. One possible reason is that Indigenous Australians are very good at remembering spatial patterns. In a version of Kim's game, where one has to recall the location of a variety objects on a tray, Kearins (1981) showed that Indigenous adolescents and children were superior to their non-Indigenous counterparts. Moreover, Kearins found that the nameability of the objects in the array to be remembered, affected non-indigenous participants but not Indigenous participants. It may well be that naming the number of objects in the array to be remembered is the preferred strategy for the English-speaking children, but not for the Northern Territory children.

Kearins (1986) considers two possible explanations for this. One is a genetic hypothesis proposed by Lockard (1971). According to this, there is selection of abilities according to niche, especially where a population is relatively isolated. Desert dwellers, of the sort that Kearins tested, are hunter-gatherers who are 'possessor of unusual knowledge and skills in the natural world. They can live off the land where almost no Westerners can do so, finding water and food in apparently arid country.' People began to occupy Australia at least 40 000 years ago (Flood, 1997) and have been relatively isolated from other populations during that time. Thus, survival in this hostile environment may have favoured those who could acquire these special skills. The ability to retain spatial and topographical information could make the difference between life and death in the desert. By contrast, the invention of agriculture 10 000 years ago put an emphasis on different kinds of skills, and also resistance to animal-originated diseases that are pandemic in Europe and Asia, such as smallpox, measles etc. (Diamond, 1997). It is striking therefore that in Kearins's study, both semi-traditional participants who lived in the desert and non-traditional participants who lived on the desert fringe performed equivalently, and better on all tasks than non-indigenous participants from a forestry and farming area. These results appear to support the genetic hypothesis since it is not where you live but your ancestry that is critical.

However, Kearins (1986) raises another possibility: differences in child-rearing practices. Indigenous Australians, like other hunter-gatherers, rarely transmit information or skills by verbal instruction ('All that nagging'). Rather children are encouraged to learn by observation. This may mean that children acquire skills of remembering what they see earlier or better than non-indigenous children. This is supported by several studies that Kearins cites. Thus, parents and the general learning environment of Indigenous Australian children encourage those skills particularly useful for the desert niche, of which good spatial memory and routine dependence on it are a part. Of course, genetic factors and child-rearing practices may not be unrelated.

We do not doubt that a good notation is helpful for carrying out mental work, in this case, carrying out simple addition. However, our results suggest that counting words are not the only good notation, and that a strategy for mapping items to be enumerated onto a spatial representation could also be effective when counting words are not available. The relationship between an accumulator mechanism and a two- or three-dimensional mental spatial array is still to be elucidated.

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