Primary students decoding mathematics tasks: The role of spatial reasoning

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A substantial body of Professor Lowrie's research is associated with spatial sense, particularly students' use of spatial skills and visual imagery to solve mathematics problems. He has co-authored *Mathematics for children: Challenging children to think mathematically* (now in its third edition) and has been the Editor of the *Australian Primary Mathematics Classroom Journal*. Professor Lowrie's current research projects include Australian Research Council grants which examine young students' ability to decode information graphics in mathematics and Mathematics in the digital age: Reframing learning opportunities for disadvantaged Indigenous and rural students.

Abstract

Representation is an important aspect of mathematics. In recent years graphics representations have become increasingly widespread as society comes to terms with the information age. Although the mathematics curricula have not varied to any recognisable degree in the past decade or so, the assessment procedures associated with mathematics education certainly have. This presentation highlights the changing nature of students' spatial reasoning as they engage with different types of mathematics representations. A case is presented which describes the shift from students' use of encoding techniques to represent mathematical ideas to an increasing reliance on students decoding graphical representations constructed by others. The presentation analyses a number of student work samples as they were videotaped completing assessment items from the National Assessment Plan for Literacy and Numeracy (NAPLAN). Implications from the study include the recognition that students need to acquire different spatial-reasoning skills which allow them to consider (and navigate) all the elements of a mathematics task, including specific features of a graphic and the surrounding text.

Introduction

Although mathematics curricula has changed little in the past ten years the way in which mathematical ideas are represented and communicated has shifted dramatically. Until recently, most mathematics tasks that primary-aged students were required to solve were heavily word based, whereas the current practice, from both curriculum and assessment perspectives, is to have more graphics embedded into task representation (Lowrie & Diezmann, 2009). This is unsurprising given the increased use of graphics in society and the increasing challenge of representing burgeoning amounts of information in visual and graphic forms. The amount of information at an individual's disposal and the extent to which this information can be manipulated and directed toward specific purposes has also increased (e.g., the detailed information available for weather forecasts). From a young age, children are exposed to visual forms of communication with more intensity and engagement, whether playing computer games, navigating web pages, or interpreting the rich design features of more traditional pictorial representations, and as a consequence different forms of sense making are required.

Within education contexts increased attention has been given to the role of representation in school mathematics (e.g., National Council of Teachers of Mathematics [NCTM] Yearbook, 2001). Mathematical representations have always been viewed as an integral component of the ideas and concepts used to understand and engage with mathematics (NCTM, 2000); however, the structure of these representations continue to evolve. In this presentation I argue that the nature and degree of influence mathematical representations have on teaching and learning contexts have changed and these changes have emerged almost unnoticed.

Representations tend to fall under two systems, namely *internal* and *external* representations. Internal representations are commonly classified as pictures 'in the mind's eye' (Kosslyn, 1983) and include various forms of concrete and dynamic imagery (Presmeg, 1986) associated with personalised, and often idiosyncratic, ideas, constructs and images. External representations include conventional symbolic systems of mathematics (such as algebraic notation or number lines) or graphical representations (such as graphs and maps).

Although these two systems do not exist as separate identities (Goldin & Shteingold, 2001), there is some scope (and benefit) for thinking of these two forms of representation in different ways. Internal representations often involve the process of encoding information. Encoding generally occurs when students construct their own representations in order to solve a task. Encoding techniques include drawing diagrams, visualising and spatial reasoning. These techniques provide students with the opportunity to understand all the elements of any given problem in a way that is meaningful to them, for example, drawing a circle and dividing it into segments in order to better understand a fraction problem. By contrast, decoding techniques are used to make sense of information within a given task, when the information has been represented visually for others to solve, for example, interpreting a map to determine the coordinate position of a specific street crossing. Ten years ago, a high proportion of mathematics tasks were word-problem based and teachers explicitly taught heuristics which included 'draw a diagram', or 'imagine the problem scene'. These approaches required encoding of information. Currently, a high proportion of tasks have a diagram embedded in the representation. As a consequence, it is hard for students to think beyond the diagram to construct representational meaning and thus approaches to problem solving now are more likely to require decoding skills.

This presentation considers the changing nature of mathematics representation in classroom practices, and an evolution in student engagement – where students are increasingly required to decode information but at the same time are less likely to experience situations in which they are challenged to encode mathematics ideas and representations.

Mandatory assessment practices, such as the National Assessment Plan for Literacy and Numeracy (NAPLAN) (MCEETYA, 2009), foster this change in student information processing. The structure and nature of NAPLAN-like tasks promote decoding, especially in situations where students are required to generate a multiple-choice solution. Our studies (e.g., Lowrie & Diezmann, 2009) have shown that students are reluctant to actually draw on their test booklets when they complete questions in the NAPLAN. Other forms of encoding, including internal representations, are seldom evoked since the answer to the questions generally appear on the page and this thus reduces the likelihood of students utilising other forms of imagery. Moreover, the types of questions posed typically require students to decode information from the graphics embedded in the task. By providing a graphical representation to scaffold thinking, a whole new set of skills and practices is brought to the fore. The capacity to interpret various forms of information is now required for students to solve tasks and these skill sets are quite different to those needed when encoding information.

Encoding and decoding information in mathematics

With colleagues I have been investigating students' encoding (Lowrie & Logan) and decoding (Diezmann & Lowrie, 2008; Lowrie & Diezmann, 2007; Logan & Greenlees, 2008) skills as they solve mathematics tasks commonly used as assessment items. The work on encoding has focused on the extent to which students utilise pictures or diagrams to make sense of tasks and the extent to which they evoke imagery to contextualise the problem. The studies that investigate students' decoding skills have considered the extent to which children make sense of information graphics that have different purpose, structure and orientation.

One of our current investigations (Lowrie & Logan) has set out to consider the influence encoding and decoding processes have on primary-aged students' mathematical thinking as they complete tasks in the NAPLAN. Grade 3 and 5 students (N = 45) who sat the 2010 NAPLAN were interviewed on the 2009 NAPLAN before attempting this year's paper. Students were videotaped as they solved the tasks and explained their solutions to ten items from the respective grade NAPLAN tests. The interview protocol encouraged the students to verbalise their thinking and to represent their thinking in ways they felt appropriate (i.e., writing down numbers or drawing a picture). The semi-structured interview allowed students the opportunity to reflect upon an experience that is otherwise only a quantitative measure of performance.

Representation and sense making with graphic-based tasks

Of the 75 items across the Grade 3 and Grade 5 tests, few items would be classified as traditional word-based problems. In fact, only 13 of the 35 Grade 3 items (37%) and 15 of the 40 Grade 5 items (38%) did not contain a graphic within the task. Moreover, only 15 items (20%) across the two tests would be considered traditional word problems. The students seldom utilised encoding skills to solve the tasks, especially internal representations like drawing a diagram and constructing personal images or representations. When students did construct such representations, they were almost entirely on tasks for which a graphic was not embedded within the task (see Figure 1). Thus, when a task contained an external graphic representation, students were unlikely to create a personalised internal representation as part of their sense making.

With regard to Figure 1, the student drew circles to represent the cakes and enclosed each group of five circles with a square to represent a box. He then proceeded to keep a tally (in his head) of the number of 'cakes' he had represented until he reached 34. He then argued that 7 boxes were required. This type of procedure represents a common encoding technique utilised by students to solve word problems.

Given the high proportion of the tasks in each test containing graphics, it was not surprising that students frequently utilised decoding techniques to solve the tasks. In these situations, the students did not have any markings and thus did not draw diagrams or pictures to scaffold their understandings. In relation to the students decoding (see Figure 2), the graphics generally had an important part to play in the task solution. In some situations, the graphic merely provided a context for the task; however, in most situations, the information contained within the graphic was indeed influential.

With regard to Figure 2, the student located the position of the library as the starting point. In order to complete the task, the student rotated the map to the right (see Figure 3) as a way of ensuring she could follow the subsequent directions. This meant she was facing the library as opposed to standing in front of the library. She then turns right along High Street, which is in fact left of the library. Consequently, she answered this task incorrectly. She had her hands on the page following the route with her fingers as she proceeded to work out the task. This example highlights the necessity of correctly decoding the graphic (in this instance a map task) in order to generate an appropriate solution.

The presentation will provide a number of examples which highlight the ways children encode and in particular, decode graphical representations in mathematics tasks.

Implications

Several practical implications emerge from the study.

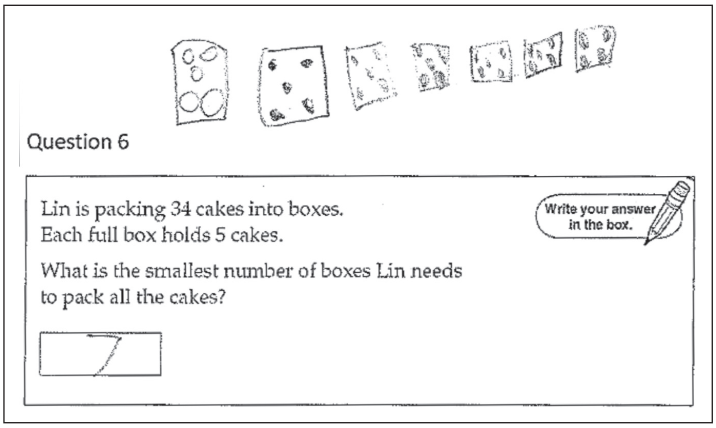

Figure 1: Example of a student using an encoding technique

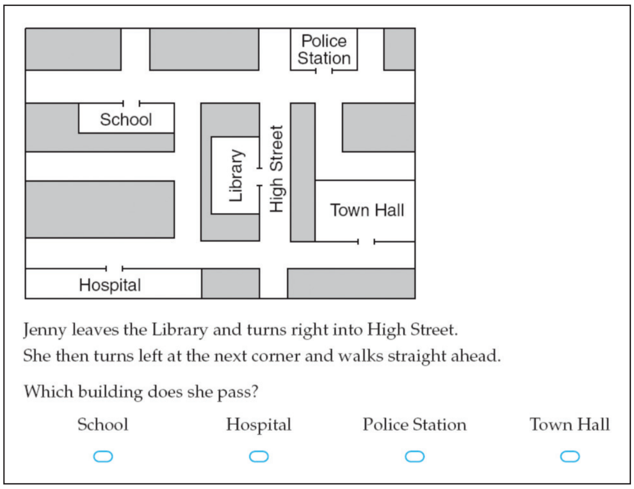

Figure 2: An example of a task that requires decoding using spatial reasoning and mental imagery

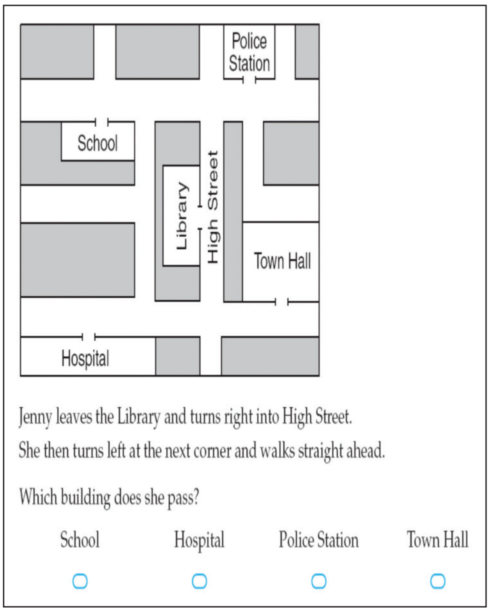

Figure 3: The same task represented in the orientation the student used to solve the item

- The movement away from traditional word-based problem solving limits students' opportunities to utilise encoding techniques to make sense of mathematics ideas. If these encoding skills are not encouraged and promoted elsewhere, students' general reasoning skills will be restricted since such techniques are necessary when students encounter novel or complex problems.
- Conversely, the introduction of mathematics tasks rich in graphics requires a different skill base. Explicit attention needs to be given to specific types of graphics since they have different structure and conventions. Teaching map-based graphics, for example, requires different approaches and techniques than graph-based graphics. Indeed bar graphs and line graphs require specific and independent attention.
- Given the increasing reliance of graphics in society, it is not surprising that graphic representations hold a prominent place in current forms of assessment. And since assessment tends to influence and even drive practice, the way in which mathematics ideas and conventions are represented impact greatly on teaching practices and student learning.
- Students are required to decode external representation with more regularity than the process of evoking internal representations through encoding. Although both require high levels of spatial reasoning, most representations are now 'teacher' generated rather than student constructed.
- Students need to acquire different spatial-reasoning skills which allow them to consider all the elements of a task, including specific features of a graphic and the surrounding text, when solving mathematics tasks.

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