The social outcomes of learning mathematics: Standard, unintended or visionary?

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Abstract

Why teach mathematics? Why should students in school learn mathematics? What are our intended aims and the outcomes of teaching and learning mathematics in school? To offer my answers to these questions I find it useful to distinguish three groups of aims/outcomes:

1 Standard aims of school mathematics – what are generally agreed to be the basic or standard reasons for teaching the subject?

2 Unintended outcomes of school mathematics – are there unexpected and unintended outcomes of the process for some or all students?

3 Visionary aims for school mathematics – what do we as mathematics educators wish to see as both aims and outcomes of school maths teaching/learning? What new emphases would enhance our students and indeed society beyond what we do now?

The standard aims of school mathematics

These are basic and functional goals that aim to develop the following capabilities:

1. Functional numeracy

This involves being able to deploy mathematical and numeracy skills adequate for successful general employment and functioning in society. This is a basic and minimal requirement for all at the end of schooling, excluding only those few with some preventative disability.

2. Practical, work-related knowledge

This is the capability to solve practical problems with mathematics, especially industry and work-centred problems. This is not necessary for all, for the depth and type of problems vary across employment types, and most occupations requiring specialist mathematics also provide specialist training. However, a strong case can be made for school providing the basic understanding and capabilities upon which further specialist knowledge and skills can be built.

3. Advanced specialist knowledge

This knowledge, learned in high school or university, is not a necessary goal for all adults, but such advanced study leads to a highly numerate professional class, as exists in France, Hungary, etc., where all students study mathematics to around 18 years of age minimum. Advanced specialist knowledge is needed by a minority of students as a foundation for a broad range of further studies at university, including STEM subjects, as well as medical and social science studies. Clearly this option must be available in an advanced technological society, and indeed more students should be encouraged to pursue it, but it should not dominate or distort the school mathematics curriculum for all.

These three categories constitute useful or necessary mathematics for all or some, primarily for the benefit of employment and society from an economic perspective, as well as sustaining mathematics and mathematical interests themselves. They also benefit the recipient students in terms of functioning in society, work and further study.

Unintended outcomes of school mathematics

What could the unintended outcomes of school mathematics be? What I have in mind are the values, attitudes and beliefs that students develop during their years of schooling that are not planned or intended, outcomes of what is known as the 'hidden' curriculum of schooling. These concern beliefs about the nature of mathematics, about what is valuable in mathematics, and about who can be successful in mathematics. These beliefs include:

- Mathematics is intrinsically difficult and inaccessible to all but a few.
- Success in mathematics is due to fixed inherited talent rather than to effort.
- Mathematics is a male domain, and is incompatible with femininity.
- Mathematics is an abstract theoretical subject disconnected from society and day-to-day life.
- Mathematics is abstract and timeless, completely objective and absolutely certain.
- Mathematics is universal, value-free and culture-free.

Every one of these beliefs is wrong, and many of my writings over the past 30 years have been devoted to showing this (Ernest 1991). The good news is that a growing number of researchers and teachers have come to reject these beliefs. Furthermore, their acceptance has always varied greatly by country and culture, so for example Asian countries typically subscribe to the belief that mathematical success is due to effort rather than intrinsic ability.

The bad news is that such beliefs are still held by many students and parents. Such beliefs are still communicated through popular images of mathematics widespread in society and the media, and in the image of mathematics presented in some classrooms.

One widespread outcome, although far from universal, is that many students develop negative attitudes about mathematics and about their own mathematical capabilities. As we have learnt from sport, attitudes are vital to success, and for students a lack of confidence in their mathematical abilities becomes a self-fulfilling prophecy – a failure cycle (Figure 1).

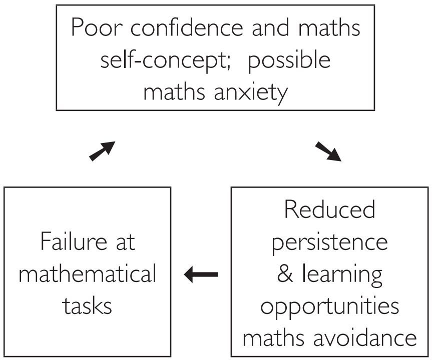

Figure 1: The failure cycle

Take another example. Despite progress, mathematics is still widely seen as a male domain, and although girls now equal boys in mathematical achievement at 16 years of age or so, too many women still doubt their own abilities and choose not to pursue mathematics related studies or careers after this age,

In my view, values, images, beliefs and attitudes about maths underlie many of the differences in learning outcomes observed across different groups of students defined in terms of sex, socio-economic status and ethnicity. For example, in Australia, mathematics performance of Indigenous Australians can lag over two years behind that of non-Indigenous students (Queensland Studies Authority, 2004). But a full account of such inequalities requires more complex explanations involving such notions as Bourdieu's cultural capital and structural inequalities present in society, as well as the maths related misconceptions discussed here.

Visionary goals for school mathematics

The traditional mathematics curriculum is defined in terms of mathematical content and its use. Instead I want to move away from content and propose aims for mathematics that are empowering and broadening for students. Students should develop:

4 Mathematical confidence

5 Mathematical creativity through problem posing and solving

6 Social empowerment through maths (critical citizenship)

7 Broader appreciation of mathematics.

These four aims are less directly utilitarian since they are more to do with personal, cultural and social relevance, although ultimately I believe they have powerful incidental benefits for society, as well as for individual students.

4. Mathematical confidence

Elevating this to an aim should come as no surprise given the importance I attach to attitudes as part of the incidental outcomes of school mathematics. Mathematical confidence includes being confident in one's personal knowledge of mathematics, feeling able to use and apply it, and being confident in the acquisition of new knowledge and skills when needed. This is the most directly personal outcome of learning mathematics, it uniquely involves the development of the whole person in a rounded way, encompassing both intellect and feelings. Effective knowledge and capabilities rest on freedom from negative attitudes to mathematics, and the feelings of enablement and empowerment, as well as enjoyment in learning and using mathematics. These latter lead to persistence in solving difficult mathematical problems, as well as willingness to accept difficult and challenging tasks. Matching but inverting the failure cycle I discussed above (see Figure 1) is the virtuous, upwardly spiralling success cycle (see Figure 2).

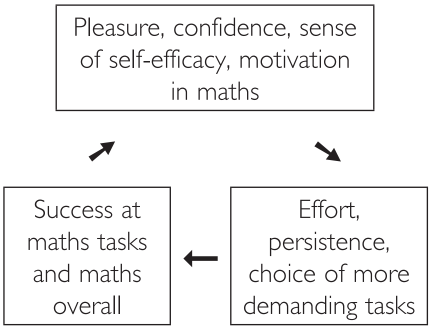

Figure 2: The success cycle

This cycle is one of the intrinsic mechanisms which draws us to the pleasures of success and self-enhancement like a light draws a moth. Indeed we can potentially turn a failure cycle into a success cycle by subtracting risk and making success achievable. In school this means reducing the importance of examinations and paying more attention to the quality of student learning experiences.

In my view this domain of attitudes, beliefs and values is one of the most important psychological dimensions of learning mathematics and we need to pay much more attention to it in school. Seemingly insignificant incidents can switch a learner on or off mathematics, and we need to be more sensitive to this in our teaching.

5. Mathematical problem posing and solving

Mathematics is too often seen as a non-creative and mechanical subject, but deploying mathematical knowledge and powers in both posing and solving problems is the area of greatest potential for creativity in school maths. Students choose which models and approaches to use in their solutions. Problem solving is widely endorsed, but too often focused on routine problems. True problem solving, the creative use of mathematics, requires non-routine problems, in which new methods and approaches must be created. Problem posing, the articulation and formulation of questions and problems to be solved, has been more neglected in maths. But it enables the seeing of mathematical connections between superficially diverse questions and topics, and the framing of questions by analogy. It involves seeking models for different aspects of life or mathematical patterns as discovered or chosen by students themselves. This is where full creativity flowers through student choices at every stage: problem or model formulation, the choice of methods to apply, and the construction of solutions.

6. Social empowerment through mathematics

Contrary to popular belief, mathematics is a political subject. Mathematics should be taught in order to socially and politically empower students as citizens in society. It should enable learners to function as numerate critical citizens, able to use their knowledge in social and political realms of activity, for the betterment of both themselves and for democratic society as a whole. This involves critically understanding the uses of mathematics in society: to identify, interpret, evaluate and critique the mathematics embedded in social, commercial and political systems and claims, from advertisements, such as in the financial sector, to government and interest-group pronouncements. Economics is applied mathematics and this is the main language of politics, power and personal functioning in society. Every citizen needs to understand the limits of validity of such uses of mathematics, what decisions it may conceal, and where necessary reject spurious or misleading claims. Ultimately, such a capability is a vital bulwark in protecting democracy and the values of a humanistic and civilised society.

Critical citizenship through mathematics is a major topic on its own and the Critical Mathematics Education movement has spring up to deal with theory and practice in this area. There are many relevant publications such as Skovsmose (1994), Ernest (2001) and the special issue of *The Philosophy of Mathematics Education Journal* forthcoming summer 2010.

7. Appreciation of mathematics

The last of my proposed seven aims or capabilities is the development of mathematical appreciation. There is an analogy between capability versus appreciation in mathematics, on the one hand, and the study of language versus that of literature, on the other. Mathematical capability is like being able to use language effectively for oral and written communication, whereas mathematical appreciation parallels the study of literature, concerned with the significance of mathematics as an element of culture and history, with its own stories and cultural pinnacles, so that the objects of mathematics are understood in this way, just as great books are in literature.

The appreciation of mathematics itself, and its role in history, culture and society in general, involves a number of dimensions and roles, including the following.

- Having a sense of mathematics as a central element of culture, art and life, present and past, which permeates and underpins science, technology and all aspects of human culture. This extends from symmetry in appreciating elements of art and religious symbolism, to understanding how modern physics and cosmology depend on algebraic equations such as Einstein's $E = mc^2$. It must include understanding how mathematics is increasingly central to all aspects of daily life and experience, through its import in commerce, economics (e.g., the stock market), telecommunications, ICT, and the role it plays in representing, coding and displaying information. However, it must be recognised that mathematics is becoming invisible as it is built into the social systems that both control and empower us in our increasingly complex societies and lives.

- Being aware of the historical development of mathematics, the social contexts of the origins of mathematical concepts, its symbolism, theories and problems. The evolution of mathematics is inseparable from the most important developments in history, from ancient societies in Mesopotamia, Egypt, India and Greece (number and tax and accounting, geometry and surveying) via medieval Europe and the Middle East (algorithms and commerce, trigonometry and navigation, mechanics and ballistics) to the modern era (statistics and agriculture-biology-medicine-insurance, logic and digital computing-media-telecommunications). This includes being aware of ethnomathematics, which studies informal culturally embedded mathematical concepts and skills from cultures around the globe, both rural and urban, past and present.
- Having a sense of mathematics as a unique discipline, with its central branches and concepts as well as their interconnections, interdependencies, and the overall unity of mathematics. This includes its central roles in many other disciplines as applied mathematics. After many years spent studying mathematics learners should have some conception of mathematics as a discipline, including understanding that there is much more to mathematics than number and what is taught in school.
- Understanding the ways that mathematical knowledge is established and validated through proof is also important, as well the limitations of proof. I believe this should include introduction to the philosophy of mathematics: understanding that there are big questions and controversies about whether mathematics is discovered or invented, about the certainty of mathematical knowledge and about what type of things mathematical objects are. Being aware of such controversies supports a more critical attitude to the social uses of mathematics, as well as withstanding attributions of certainty to anything mathematical.
- Learners should gain a qualitative and intuitive understanding some of the big ideas of mathematics such as pattern, symmetry, structure, proof, paradox, recursion, randomness, chaos, infinity. Mathematics contains many of the deepest, most powerful and exciting ideas created by humankind. These extend our thinking and imagination, as well as providing the scientific equivalent of poetry, offering noble, aesthetic, and even spiritual experiences.

Are these aims concerning appreciation feasible for school? Even big ideas like infinity can be appreciated by schoolchildren. Many an interested 8-year-old will happily discuss the infinite size of space, or the never-ending nature of the natural numbers.

In mathematics we are privileged to have around 2000 hours of compulsory school time over the years – surely we can afford to spend some time on these visionary aims – they have the potential to help build more confident and knowledgeable students and citizens, and dare I say it, a better society?

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