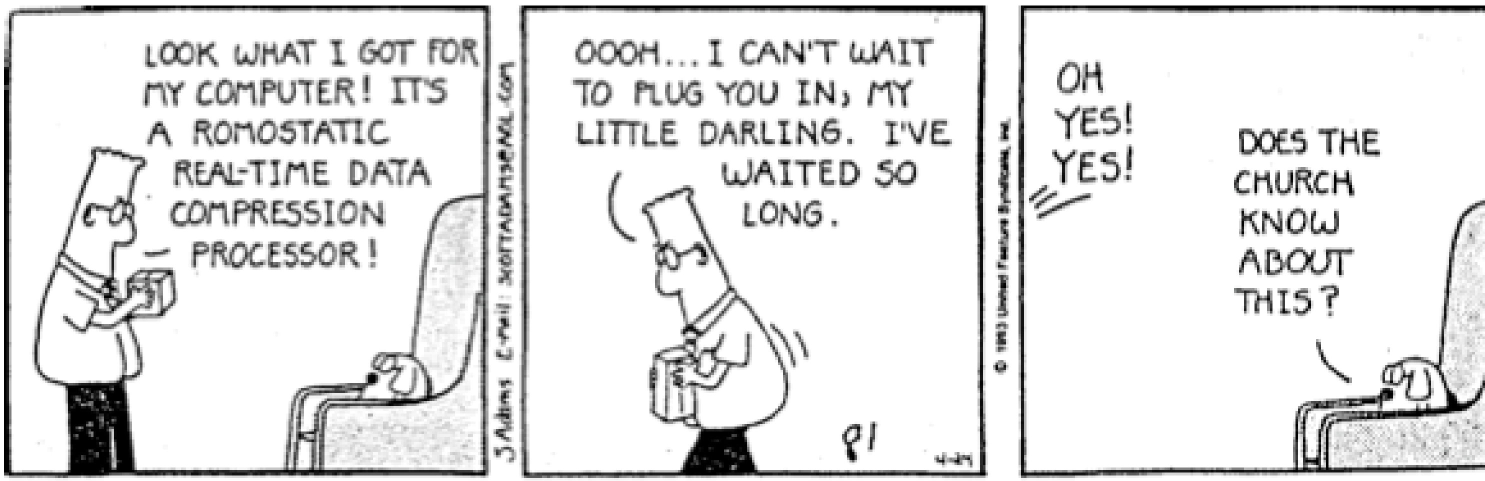
Using technology to support effective mathematics teaching and learning: What counts?

Merrilyn Goos

The University of Queensland

First, a confession …



Which technologies?

- Hand held devices
  - Four function calculators
  - Scientific calculators
  - Graphics calculators
  - CAS-enabled graphics calculators
  - Interactive devices (e.g., Casio ClassPad, TI-Nspire)
  - PDA devices (e.g., iPod Touch, iPhone, iPad)
- Computer software
  - Spreadsheets
  - Dynamic geometry (Cabri, Geometer's SketchPad, Deogebra)
  - Statistics (Fathom, TinkerPlots)
- Internet
  - Web resources and applications
  - Social networking, Web 2.0 etc

(Kissane, 2010)

A question to ponder

What is the purpose of using digital technologies for learning and teaching mathematics?

- To get "the answer" more quickly and accurately?
- To supplement "by hand" skills? (e.g., graphing)
- To improve (and even change) the way students learn mathematics?

A plan

1. Messages from research on learning and teaching mathematics with digital technologies.
2. Snapshots of practice – what can effective classroom practice look like?
3. Technology messages in the draft *Australian curriculum – Mathematics*.

Key messages from research

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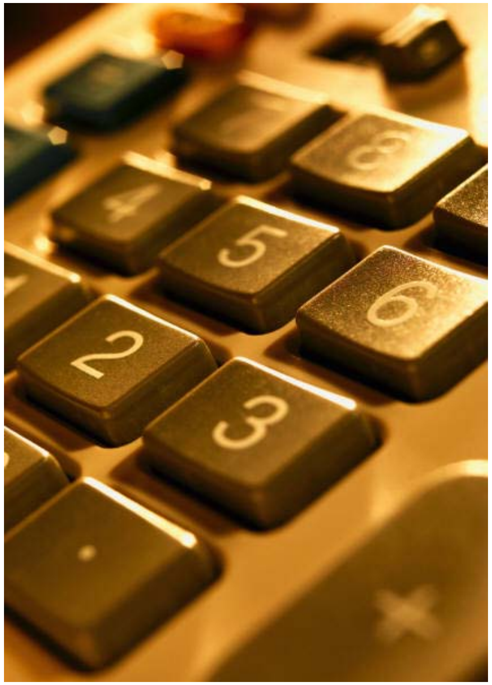
A common research question …

What **effects** do calculators have on:

•students' mathematical **achievement**?

ooperational, computational, conceptual, problem solving skills?

•students' **attitudes** towards mathematics?



Key messages from research

Summary of findings (Ellington, 2003)

Achievement

How calculators are used	Op	Comp	Conc	PS
Teaching only	⇧	–	–	–
Teaching + assessment	⇧	⇧	⇧	⇧

Attitudes

Calculator use > non-calculator use

Key messages from research

A more interesting research question …

What **changes** when students and teachers have access to technology?

- Mathematical knowledge
- Mathematical practices

Key messages from research

Technology and mathematical knowledge

If one considers mathematics to be a **fixed body of knowledge** to be learned, then the role of technology in this process would be primarily that of an **efficiency tool**, i.e. helping the learner to do the mathematics more efficiently. However, if we consider the technological tools as providing **access to new understandings** of relations, processes, and purposes, then the role of technology relates to a **conceptual construction kit**. (Olive & Makar, 2010, p. 138)

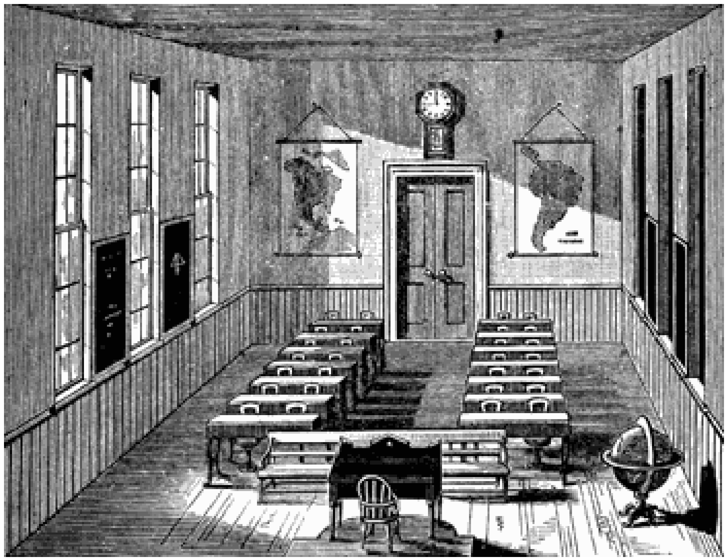
Key messages from research

Technology and mathematical knowledge

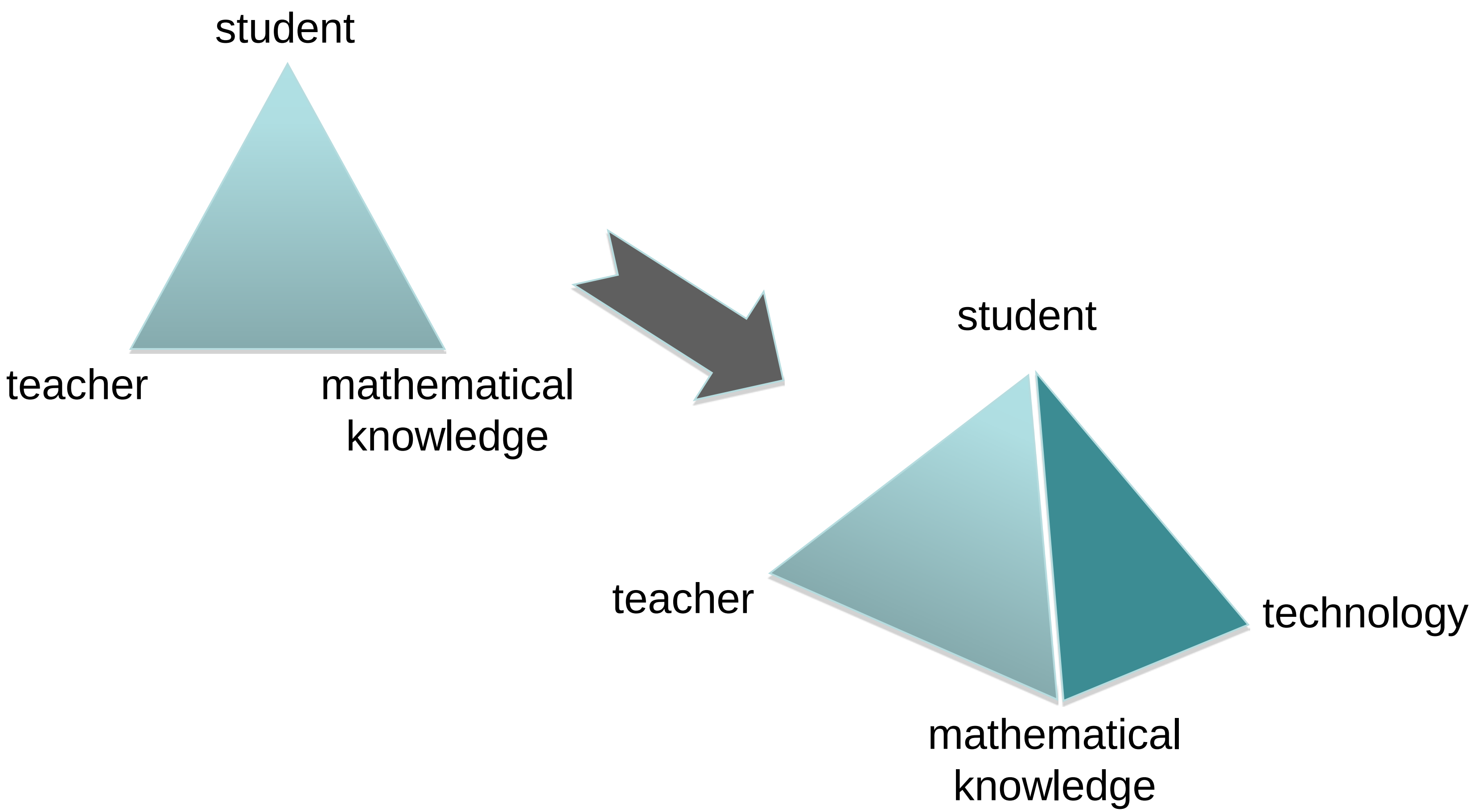
- An example of knowledge generation using Cabri Geometry

Key messages from research

- **Technology and mathematical practices**

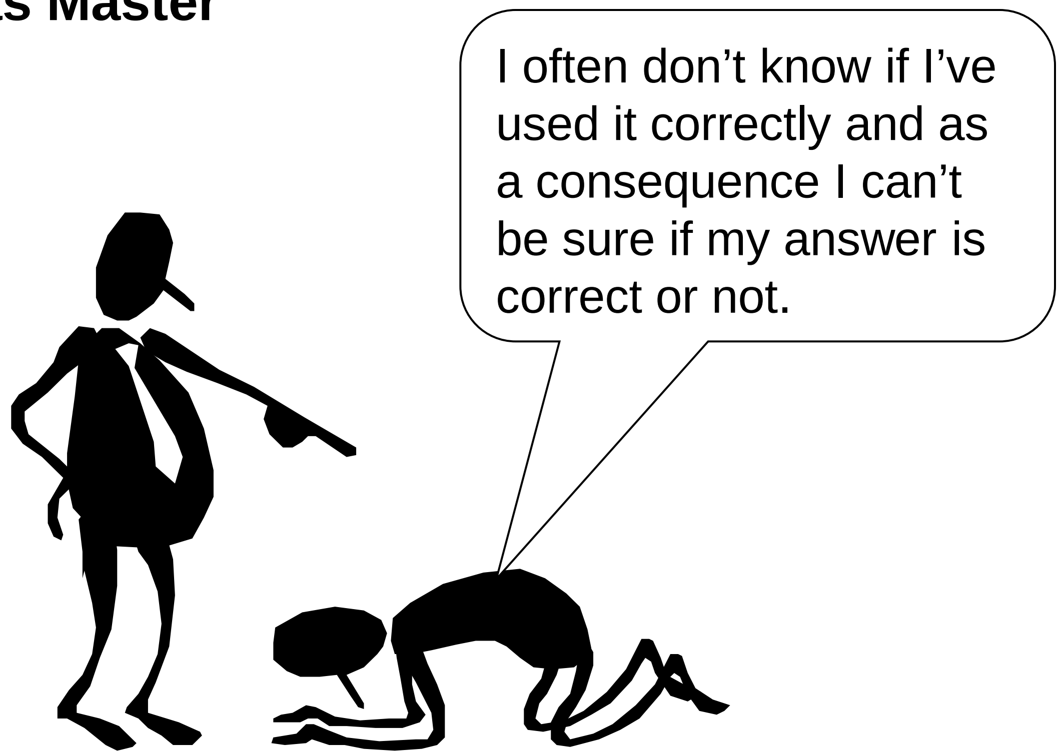


Key messages from research



Technology and mathematical practices

Technology as Master



Technology and mathematical practices

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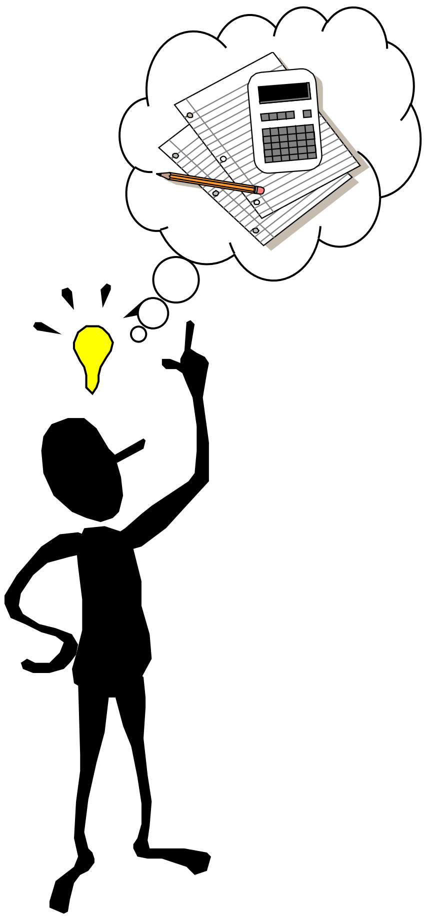
Technology as Servant

I regularly use technology for *familiar* tasks purely as a time saver and to verify and check my answers.

Technology as Partner

"My calculator has become my best friend. His name is Wilbur. Me and Wilbur go on fantastical adventures together through Maths land. I don't know what I'd do without him."



Technology as Extension of Self

“My calculator is practically a part of myself. It’s like my third brain. I use it whenever it can help me do anything faster.”

Technology and mathematical practices

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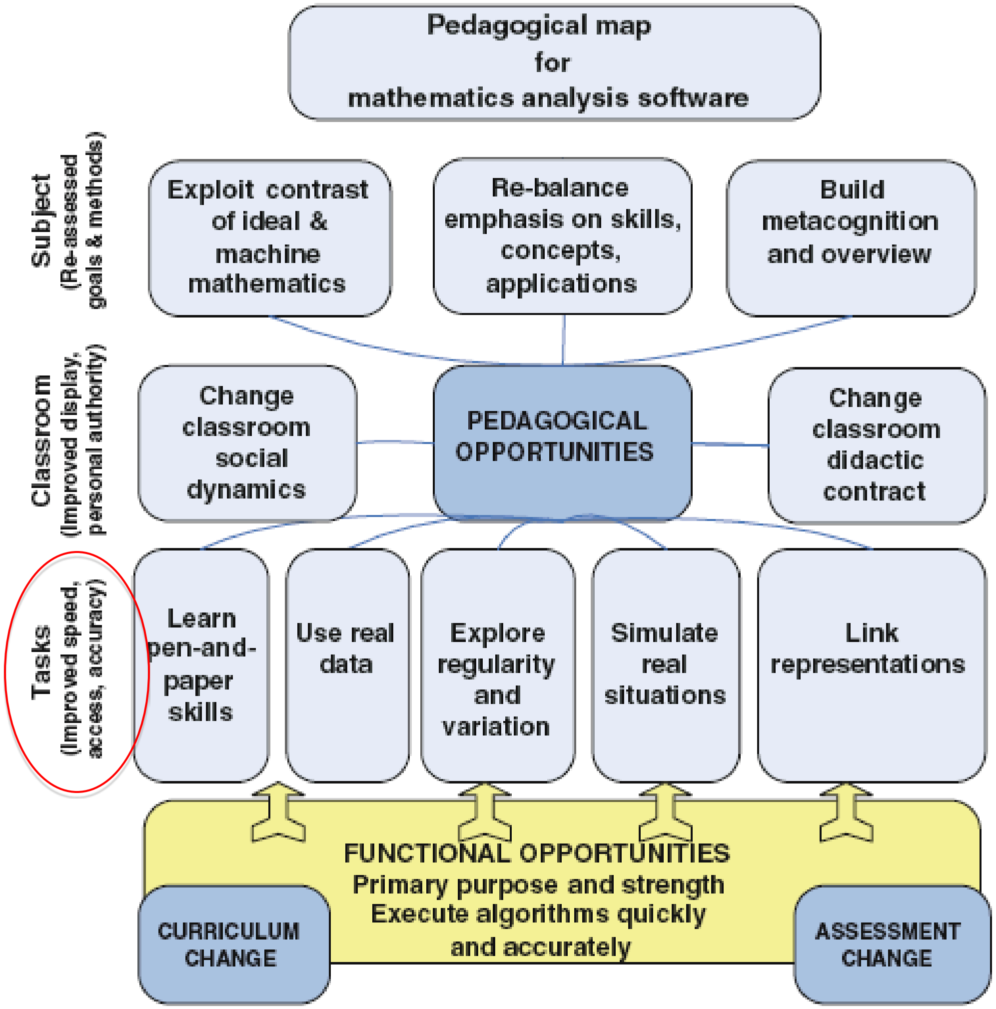
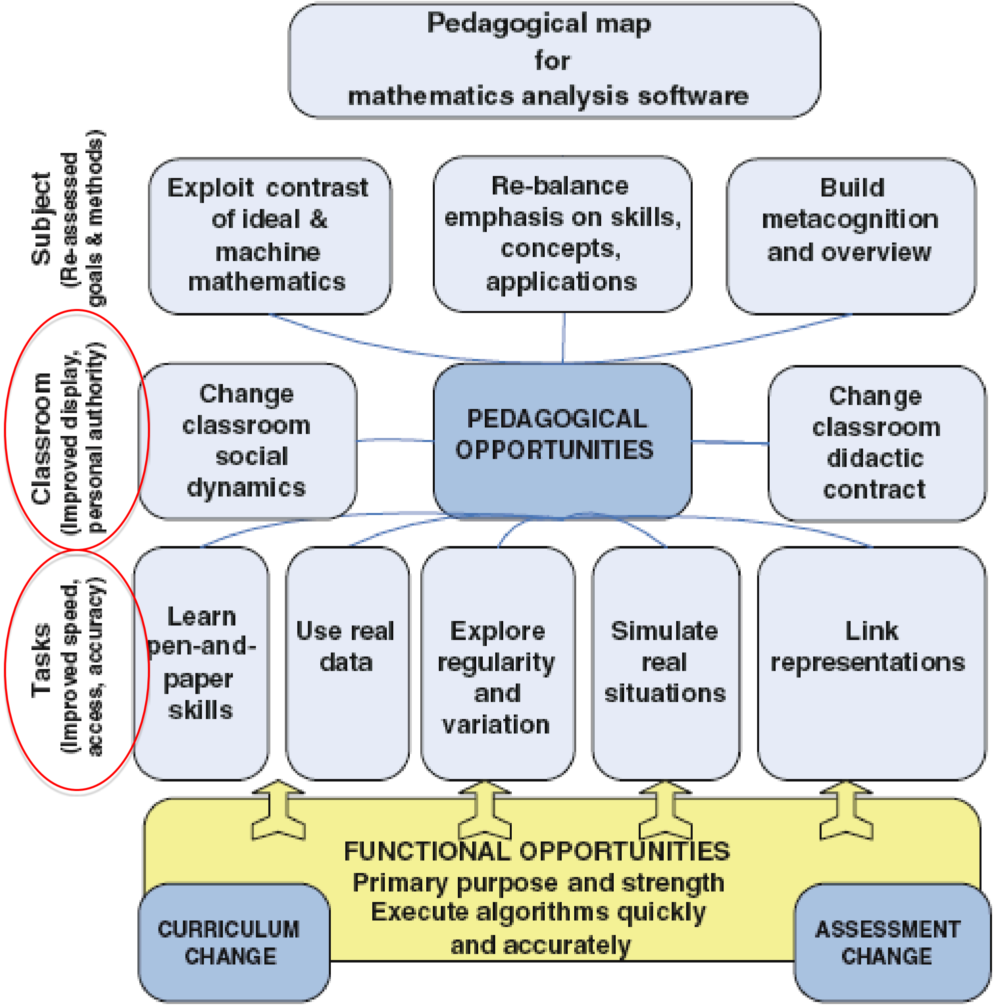
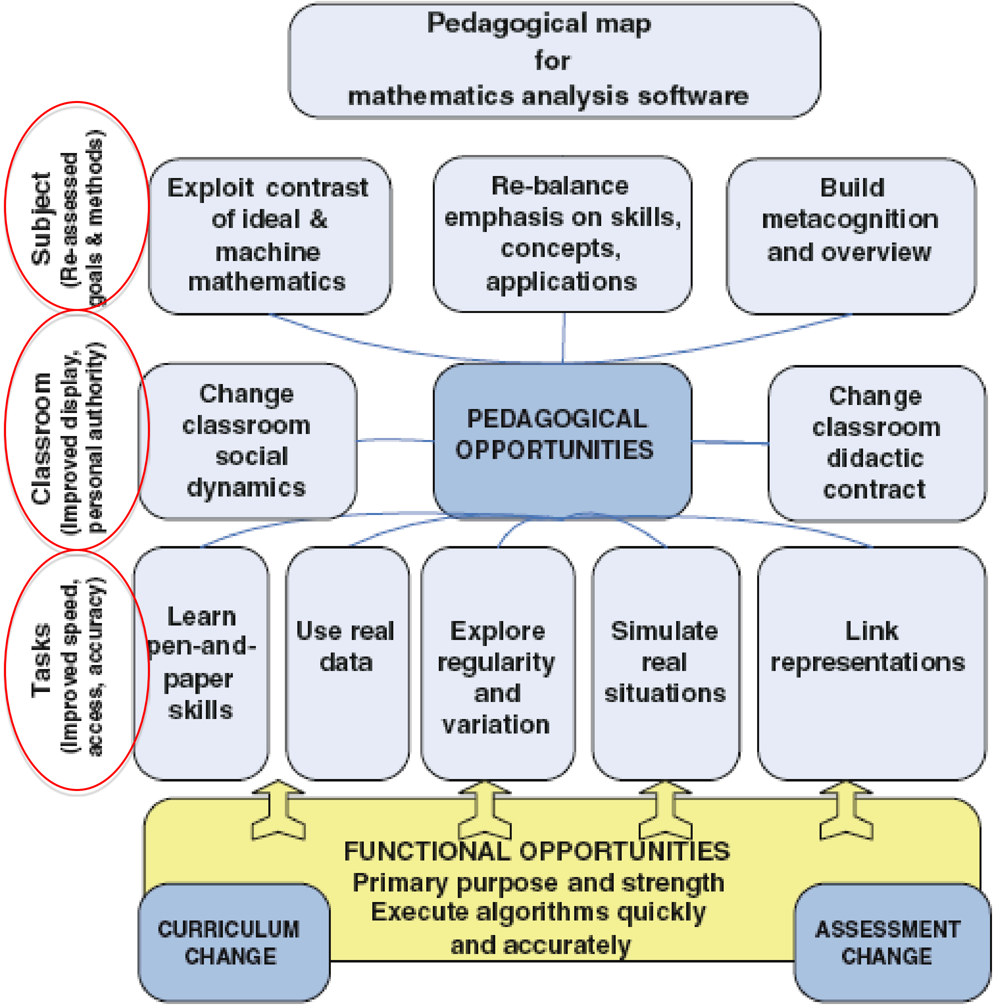


100 YEARS AHEAD

Technology and mathematical practices



Technology and mathematical practices

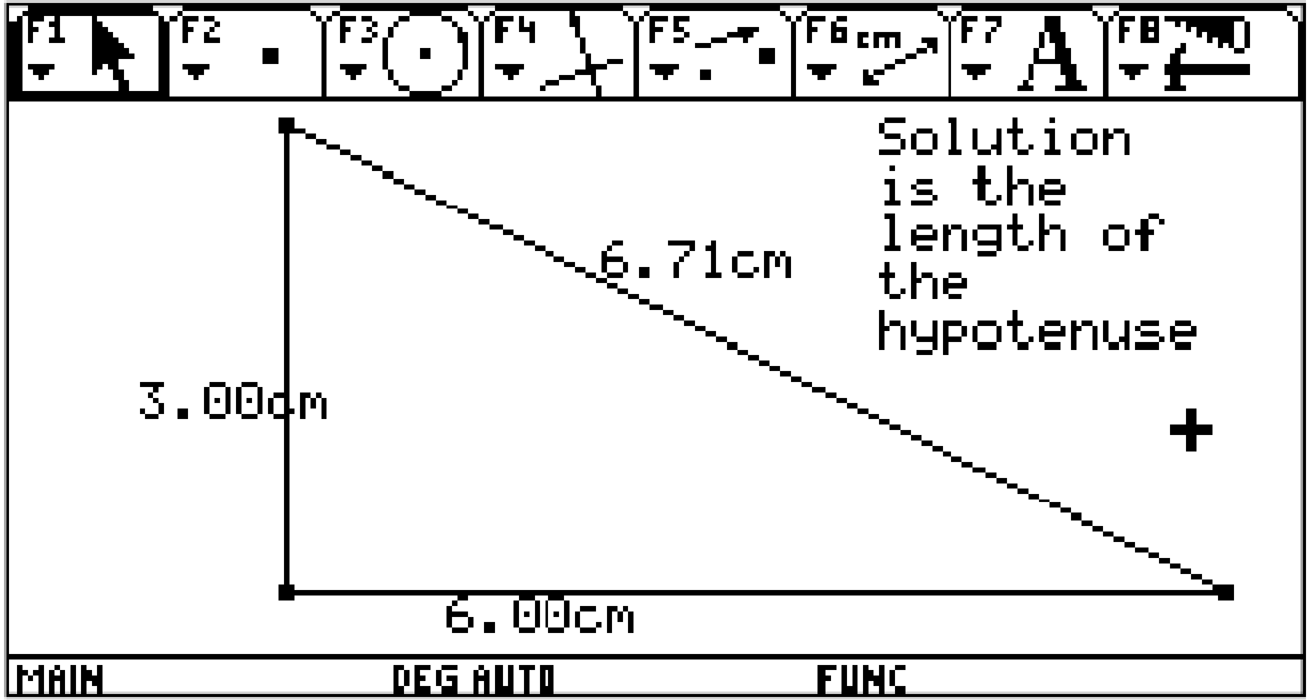


Snapshot #1 of practice

Draw a line $\sqrt{45}$ units long.

Snapshot #1 of practice

Draw a line √45 units long.



$$3^2 + 6^2 = \left(\sqrt{45}\right)^2$$

Snapshot #1 of practice

Episode 1

Students find the square root of various numbers. **(Servant)**

$$\sqrt{45} = 6.7082039...$$

Students pass calculators back and forth to share and critique each other's thinking. **(Partner)**

Snapshot #1 of practice

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Episode 2

Teacher hint: Think about triangles

Students search for Pythagorean triples without geometric representation. **(Servant)**

Teacher: But what would it *look* like?

Sam: Well, you could have a triangle …

Teacher: (to class) I think Sam's given you a hint!

Nicole: Has it got something to do with Pythagoras?

Teacher: Way to go!

Snapshot #1 of practice

Episode 2 (cont'd)

Susie: So we're trying to relate it to 45! The hypotenuse.

Nicole: So the side of "a" could be √5 and the side of "b" could be √40, so √40 squared and √5 squared is 45.

Susie: But the length of the sides will be an irrational number.

Snapshot #1 of practice

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Episode 2 (cont'd)

Teacher: There seems to be a lot positively related to the work we were doing yesterday, but walking around, there were five of you doing geometry and the rest of you were on your calculators working only with numbers. So, some of us are going to have to take a little risk and get out of our comfort zones. We like working with numbers because it's comfortable, but just because you're busy doesn't mean it's productive. Other people have given you big hints. You need to try to work with that.

Snapshot #1 of practice

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Episode 2 (cont'd)

Adam: You can't add two roots together.

Nicole: Yes it does! It works! You square it. You square it so it gets rid of the square root, so it ends up with 40 and 5. But if you end up with 20 and 25, then you can do √20 is 2√5, and √25 is 5, so that's squared.

Adam: Is that it? Is that √5?

Nicole: Ahhh! I just feel so relieved! It just works! It really does work!

Snapshot #1 of practice

Episode 2 (cont'd)

Adam: We worked it out on that! (shows calculator to teacher)

Teacher: What are you doing with that?

Adam: Because we didn't have the calculator.

Teacher: But I said stop playing with that! You should be in geometry!

Nicole: What are we supposed to be doing? Geometry?

Snapshot #1 of practice

Episode 3

Student proposes geometric solution, passes his calculator around group to share and defend his solution. **(Partner)**

Diane: We’ve got two that are the same, So 45 = 9 + 36.

Gena: Yeah, so that equals 9. So write that down!

Karen: Tom wants your attention.

Gena: What are you trying to say Tom?

Tom: No. because that squared plus that squared … whatever the number what … 6 point something … so ... To get that number, you need a right-angled triangle with a side of 6 and 3 and that -

Snapshot #1 of practice

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Episode 3 (cont'd)

Frances: I get it now!

Harry: How do you know it's going to be a rational number?

Tom: That! That's the root of 45, If you want to know how to draw it -

Harry: That line is going to be …

Tom: By drawing that and that and the right angle, you get that.

Gena: Yay! Cool!

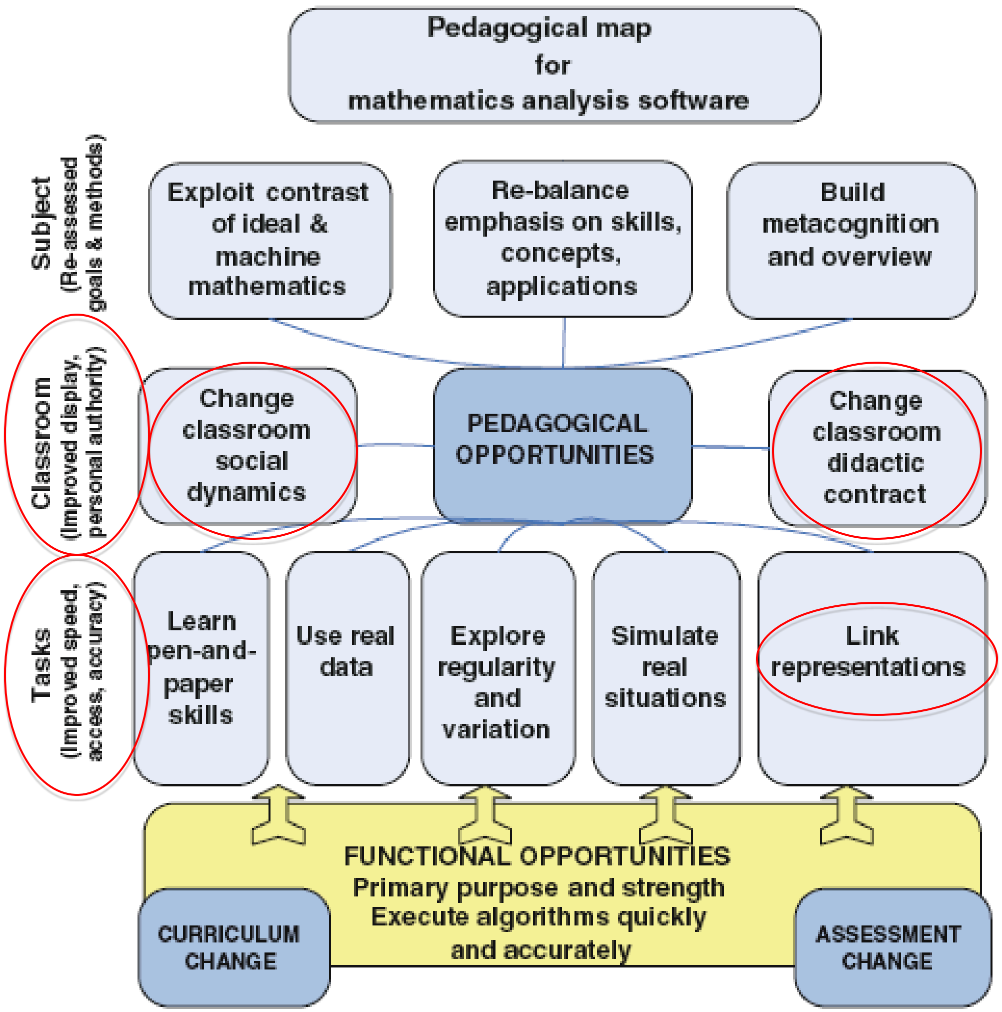
Snapshot #1 of practice

Episode 3 (cont'd)

Diane: So Tom, this has to be 3 and that has to be 6, so it ends up equalling the 6 point something number.

Tom: √36 squared plus √9 squared equals 45, and 6 squared plus 3 squared is 45.

Find a line $\sqrt{45}$ units long



Snapshot #2 of practice

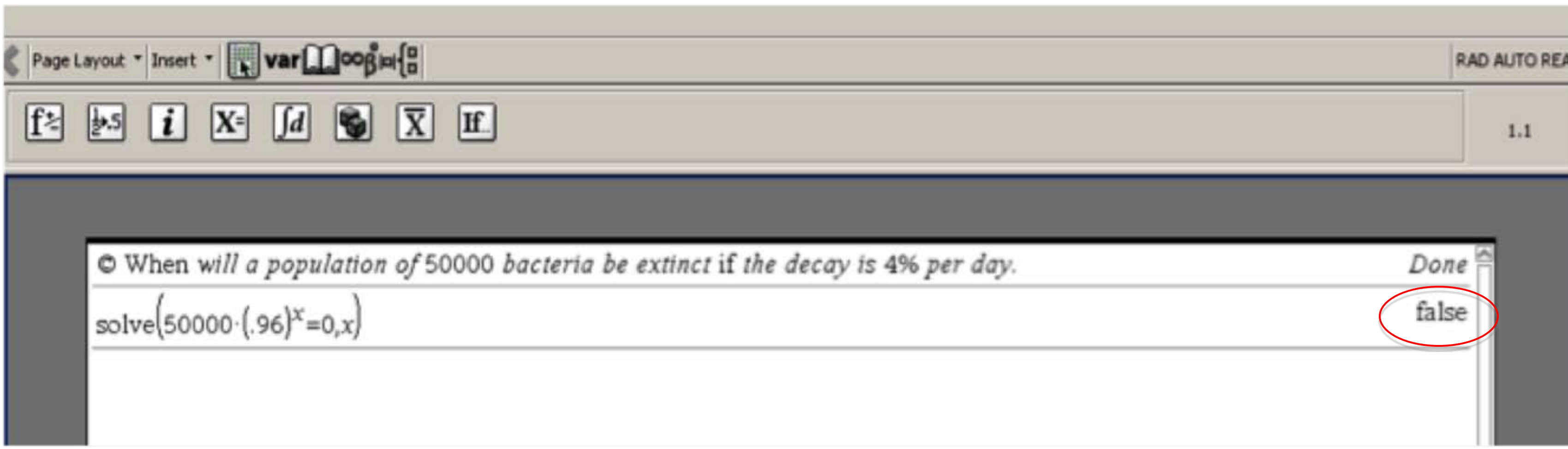
When will a population of 50,000 bacteria become extinct if the decay rate is 4% per day?

Snapshot #2 of practice

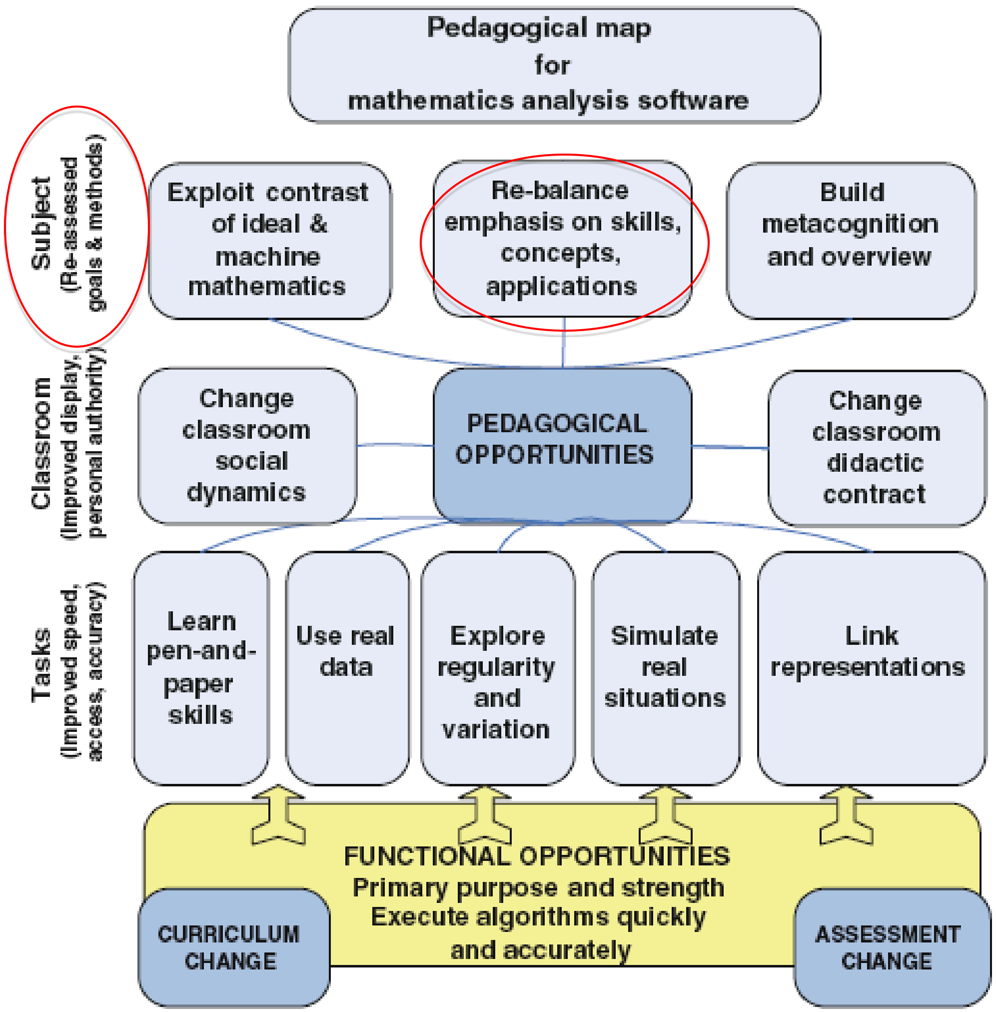
When will a population of 50,000 bacteria become extinct if the decay rate is 4% per day?

$$y = 50000 \times (0.96)^x$$

Snapshot #2 of practice



Finding an exponential model



Technology in the curriculum

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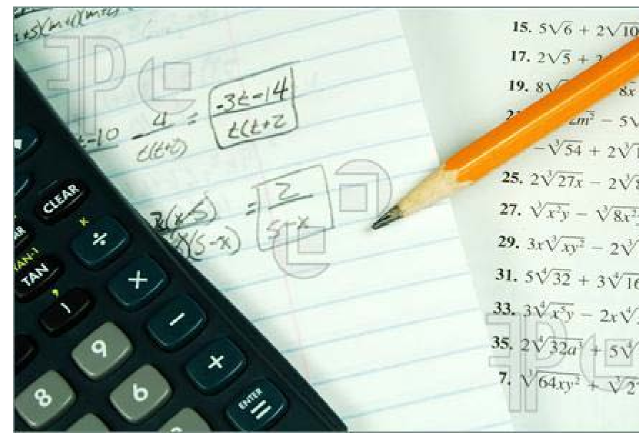
National Council of Teachers of Mathematics *Principles and Standards for School Mathematics* (2000).

The Technology Principle

Technology is essential in teaching and learning mathematics;
it influences the mathematics that is taught and enhances students' learning.



Technology in the curriculum

Australian Association of Mathematics Teachers *Communique on graphics calculators and school mathematics* (2000)

There is a compelling case for the advantages offered to students who use graphics calculators when learning mathematics. They are empowering learning tools, and their effective use in Australia's classrooms is to be highly recommended.

Technology in the curriculum

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Shape of the Australian curriculum: Mathematics (May 2009)

An important consideration in the structuring of the curriculum is to embed digital technologies so that they are note seen as optional tools. Digital technologies allow new approaches to explaining and presenting mathematics as well as assisting in connecting representations and thus deepening understanding.

Digital technologies can make previously inaccessible mathematics accessible, and enhance the potential for teachers to make mathematics interesting to more students, including the use of realistic data and examples.

Technology in the curriculum

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K-10 mathematics curriculum draft version 1.0

It is expected that mathematics classrooms will make use of all available ICT in teaching and learning situations.

(four function calculators, graphical and CAS calculators, spreadsheets, dynamic geometry programs, other software)

Technology in the curriculum

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Mathematics

Year 7 Content descriptions	Year 8 Content descriptions	Year 9 Content descriptions
Number and Algebra	**Number and Algebra**	**Number and Algebra**
1. Indices	**1. Ratio and rate**	**1. Financial maths**
Understand and work fluently with index notation and represent whole numbers as a product of powers of prime numbers (M7NA1)	Solve problems involving use of percentages, rates and ratios, including percentage increase and decrease and the unitary method and judge reasonableness of results (M8NA1)	Solve problems in financial mathematics including applications of simple and compound interest including using ICT and judge reasonableness of results (M9NA1)
2. Integers	**2. Index laws**	**2. Index laws**
Order, add and subtract integers fluently and identify patterns for multiplication and division including using ICT (M7NA2)	Understand, describe and use generalisations of the index laws with positive integral indices (M8NA2)	Work fluently with index laws, in both numeric and algebraic expressions and use scientific notation, significant figures and approximations in practical situations (M9NA2)
3. Calculation	**3. Calculation**	**3. Linear and quadratic functions**
Understand and become fluent with written, mental and calculator strategies for all four operations with fractions, decimals and percentages (M7NA3)	Solve problems involving fractions, decimals and percentages, including those requiring converting and comparing, and judge the reasonableness of results using techniques such as rounding (M8NA3)	Understand simplification techniques for linear and quadratic functions including collecting like terms, common factors, the expansion of binomial products and simple binomial factorisation (M9NA3)
4. Variables	**4. Algebra**	**4. Linear equations**
Apply the associative, commutative and distributive laws and the order of operations to mental and written computation and generalise these processes using variables (M7NA4)	Generalise the distributive law to expansion and factorisation of simple algebraic expressions and use the four operations with algebraic expressions (M8NA4)	Solve problems involving linear equations and inequalities and substitution into, and rearrangement of formulas (M9NA4)
5. Linear equations	**5. Linear equations**	**5. Simultaneous equations**
Use symbols to represent linear relationships and solve problems involving linear relationships where there is only one occurrence of a variable (M7NA5)	Create, solve and interpret linear equations, including those using realistic contexts using algebraic and graphical techniques (M8NA5)	Solve problems involving linear simultaneous equations, using algebraic and graphical techniques including using ICT (M9NA5)
6. Coordinates	**6. Coordinates**	
Plot points on the Cartesian plane using all four quadrants (M7NA6)	Plot graphs of linear functions and use these to find solutions of equations including using ICT (M8NA6)	

100 YEARS AHEAD

Technology in the curriculum

Senior mathematics courses draft version 1.1.0

The *Shape of the Australian curriculum – Mathematics* states that available technology should be used for teaching and learning situations. Technology can include computer algebra systems, graphing packages, financial and statistical packages and dynamic geometry. These can be implemented through either a computer or calculator.

Technology can aid in developing skills and allay the tedium of repeated calculations.

Technology in the curriculum

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Senior mathematics courses draft version 1.1.0

Variable messages across courses about the role of technology:

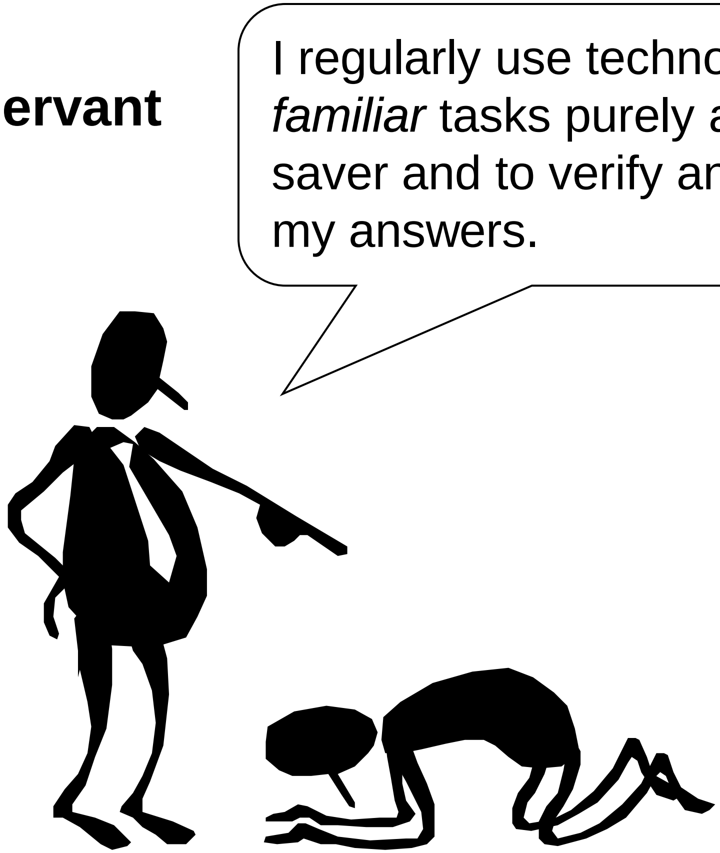
- *Assumed*, *Vital* (courses A & B);
- *Should be widely used in this topic*, *Can be used to illustrate practically every aspect of this topic* (courses C & D);
- No mention at all in some topics (courses C & D)

Implication is that technology is only valuable for less able students and only for getting “the answer”.

Technology in the curriculum

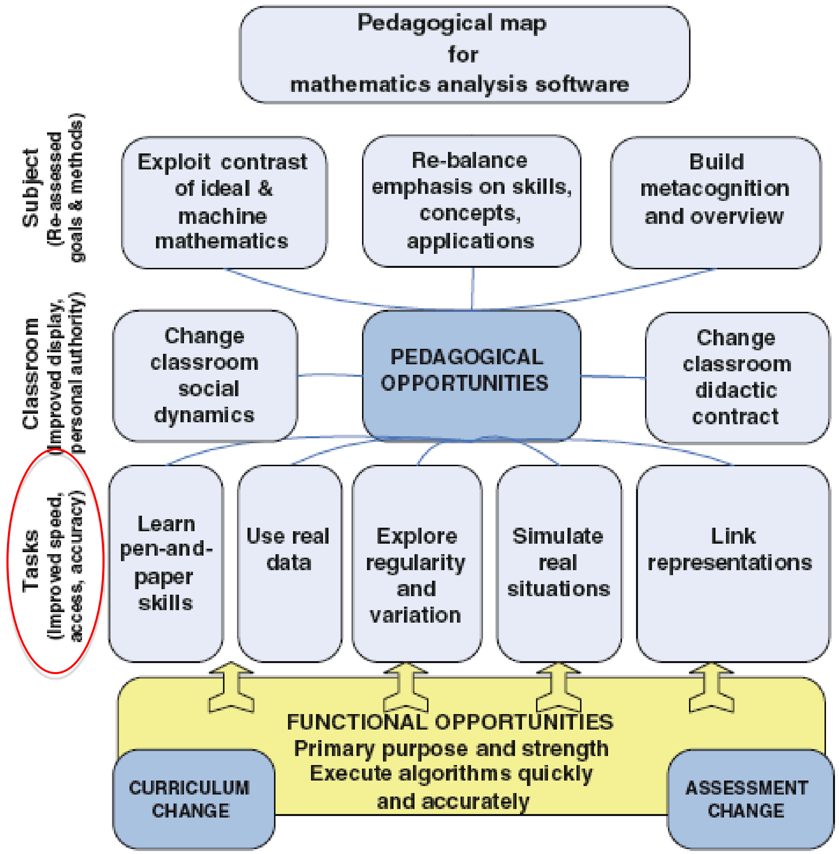
How is technology represented in the *Australian curriculum – Mathematics*?

Technology as Servant



Technology in the curriculum

How is technology represented in the *Australian curriculum – Mathematics*?



It's the enacted curriculum that counts