Curriculum Standards and Differences: lessons learned in the U.S.

corestandards.org

Phil Daro August 1, 2010

COMMON CORE STATE STANDARDS FOR

Mathematics

COMMON CORE
STATE STANDARDS INITIATIVE
PREPARING AMERICA'S STUDENTS FOR COLLEGE & CAREER

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Differences

- Standards mean “same for all”
- That is why I am going to discuss two ‘differences’ today:

1. Differences students bring to instruction
2. Differences as the metric for achievement

Curriculum Standards are meant to *reduce* unwanted differences

…not just deduce them.

Curriculum Standards can function as a platform for managing or, as easily, mismanaging instruction

Standards and their offspring, assessments, motivate or demoralize the system; that is, the people in the system: students, teachers, and their friends.

What's at stake in the design of standards based systems: Motivate or demoralize.

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Motivate or demoralize.

Why do students have to do math problems?

Why do students have to do math problems?

1. to get answers because Homeland Security needs them, pronto
2. I had to, why shouldn’t they?
3. so they will listen in class
4. to learn mathematics

Why give students problems to solve?

To learn mathematics.

Answers are part of the process, they are not the product.

- The product is the student's mathematical knowledge and know-how.
- The 'correctness' of answers is also part of the process. Yes, an important part.

Wrong Answers

- Are part of the process, too
- What was the student thinking?
- Was it an error of haste or a stubborn misconception?

Three Responses to a Math Problem

1. Answer getting
2. Making sense of the problem situation
3. Making sense of the mathematics you can learn from working on the problem

Answers are a black hole: hard to escape the pull

- Answer getting short circuits mathematics, making mathematical sense
- Very habituated in US teachers versus Japanese teachers
- Devised methods for slowing down, postponing answer getting

Answer getting vs. learning mathematics

- USA:

How can I teach my kids to get the answer to this problem?

Use mathematics they already know. Easy, reliable, works with bottom half, good for classroom management.

- Japanese:

How can I use this problem to teach the mathematics of this unit?

Teaching against the test

3 + 5 = [ ]

3 + [ ] = 8

[ ] + 5 = 8

8 - 3 = 5

8 - 5 = 3

N stands for the number of hours of sleep Ken gets each night. Which of the following represents the number of hours of sleep Ken gets in 1 week?

Ⓐ $N + 7$

Ⓑ $N - 7$

Ⓒ $N \times 7$

Ⓓ $N \div 7$

SOURCE: U.S. Department of Education, National Center for Education Statistics, *National Assessment of Educational Progress (NAEP) 2005 Mathematics Assessment*, Grade 4, Block Z1M12 #12, 2005.

Anna bought 3 bags of red gumballs and 5 bags of white gumballs. Each bag of gumballs had 7 pieces in it. Which expression could Anna use to find the total number of gumballs she bought?

A (7 X 3) + 5

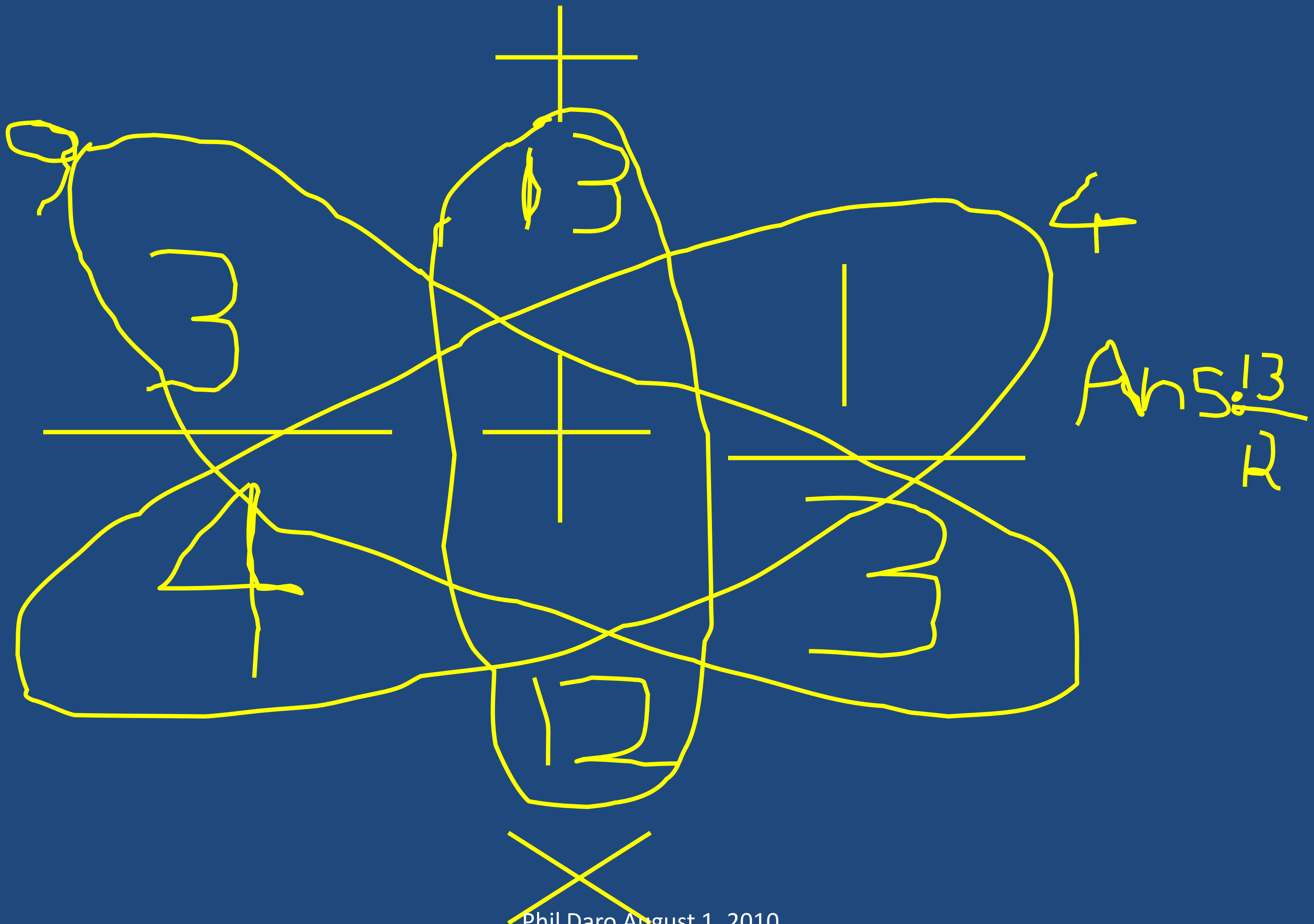
B (7 X 5) + 3

C 7 X (5 + 3)

D 7 + (5 X 3)

Butterfly method

$$\frac{3}{4} + \frac{1}{3}$$


13
3
1
4
3
12
Ans: 13/2

Use butterflies on this TIMSS item

1/2 + 1/3 +1/4 =

What mathematics do we want students to learn from work on this problem …

- Sasha went 60 miles at 12 mph. How long did it take?

that they can use on this problem?

- Xavier went 85 miles in two and a half hours,
  a) how fast was he going?
  b) At this speed, how long would it take to go *d* miles?
  c) How far could he go in *t* hours?

Set up

- Not:
  - "set up a proportion and cross multiply"
- But:
  - Set up an equation and solve
- Prepare for algebra, not just next week's quiz.

Write 3 word problems for $y = rx$, where r is a rate.

a) When r and x are given
b) When y and x are given
c) When y and r are given

Foil FOIL

- Use the distributive property
- It works for trinomials and polynomials in general
- What is a polynomial?
- Sum of products = product of sums
- This IS the distributive property when “a” is a sum

Answer Getting

ANSWER GETTING

Getting the answer one way or another and then stopping

Learning a specific method for solving a specific kind of problem (100 kinds a year)

Answer Getting Talk

- Wadja get?
- Howdja do it?
- Do you remember how to do these?
- Here is an easy way to remember how to do these
- Should you divide or multiply?
- Oh yeah, this is a proportion problem. Let’s set up a proportion?

Canceling

$x^5/x^2 = x \bullet x \bullet x \bullet x \bullet x \,/\, x \bullet x$

$x^5/x^5 = x \bullet x \bullet x \bullet x \bullet x \,/\, x \bullet x \bullet x \bullet x \bullet x$

Misconceptions about misconceptions

- They weren't listening when they were told
- They have been getting these kinds of problems wrong from day 1
- They forgot
- The other side in the math wars did this to the students on purpose

More misconceptions about the cause of misconceptions

- In the old days, students didn't make these mistakes
- They were taught procedures
- They were taught rich problems
- Not enough practice

Maybe

- Teachers' misconceptions perpetuated to another generation (where did the teachers get the misconceptions? How far back does this go?)
- Mile wide inch deep curriculum causes haste and waste
- Some concepts are hard to learn

Whatever the Cause

- When students reach your class they are not blank slates
- They are full of knowledge
- Their knowledge will be flawed and faulty, half baked and immature; but to them it is knowledge
- This prior knowledge is an asset and an interference to new learning

Second grade

- When you add or subtract, line the numbers up on the right, like this:
- 23
- +9

- Not like this
-   23
- +9

Third Grade

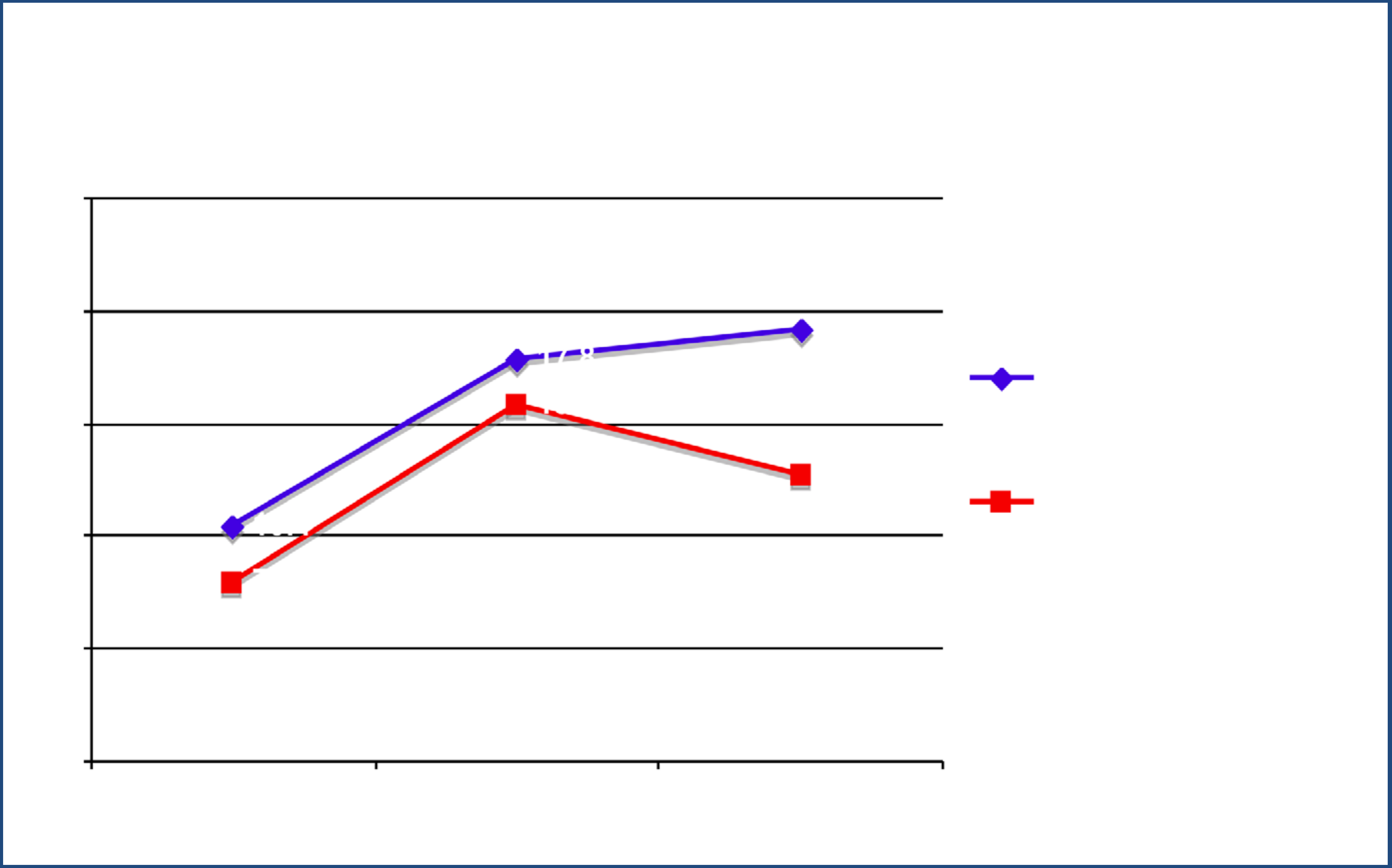
- 3.24 + 2.1 = ?
- If you "Line the numbers up on the right " like you spent all last year learning, you get this:

- 3.2 4
- <u>+ 2.1</u>
- <u>You get the wrong answer doing what you learned last year. You don't know why.</u>
- <u>Teach: line up decimal point.</u>
- Continue developing place value concepts

Stubborn Misconceptions

- Misconceptions are often prior knowledge applied where it does not work
- To the student, it is not a misconception, it is a concept they learned correctly…
- They don't know why they are getting the wrong answer

Research on Retention of Learning: Shell Center: Swan et al



The Next Step

- People are the next step
- If people just swap out the old standards and put the new in the old boxes and powerpoints
- Put them into old systems and procedures
- Put them into the old relationships

- Then nothing will change

Standards are a platform for instructional systems

- This is a new platform for better instructional systems and better ways of managing instruction
- Builds on achievements of last 2 decades
- Builds on lessons learned in last 2 decades
  - Lessons about time and teachers

Old Boxes

- “Alignment”, “covering standards” and “pacing” belong to a well intended, but weak concept for standards based teaching and learning.
  - Alignment is a bunt, lucky if you get to first base. We have to score. We need to swing at the ball. Cheap links.
  - Covering standards = mile wide-inch deep
  - Pacing means keep turning pages regardless of what students are learning: ignore student results.

It is time to move on to something stronger, more effective.

Common Core State Standards are designed as a tool to *raise* achievement, not just praise it.

- Cleared out the clutter from the basement and attic of the curriculum
- Depth, focus and coherence

Grain size is a major issue

- Mathematics is simplest at the right grain size.
- “Strands” are too big, vague e.g. “number”
- Lessons are too small: too many small pieces scattered over the floor, what if some are missing or broken?
- Units or chapters are about the right size (8-12 per year)
- STOP managing lessons,
- START managing units

Teachers should manage lessons

- Lessons take one or two days or more depending on how students respond
- Yes, pay attention to how they respond
- Each lesson in the unit has the same learning target which is a cluster of standards
- “what mathematics do I want my students to walk away with from this chapter?”

Social Justice

- Main motive for standards
- Get good curriculum to all students
- Start each unit with the variety of thinking and knowledge students bring to it
- Close each unit with on-grade learning in the cluster of standards

Standards are a peculiar genre

1. We write as though students have learned approximately 100% of what is in preceding standards. This is never even approximately true.
2. Standards are high points, finish lines, not complete specs for curriculum.
3. The grain size of coherence in mathematics is larger than "standards-based management systems" assume: Chapters or units are mathematically more coherent than lessons.

The most important ideas in the CCSS mathematics that need attention.

1. Properties of operations: their role in arithmetic and algebra
2. Mental math and [algebra vs. algorithms]
3. Units and unitizing
4. Operations and the problems they solve
5. Quantities-variables-functions-modeling
6. Number-Expression-equation-function
7. Modeling
8. Practices

Properties

Learn to be more fluent in the language of mathematics

Nine properties are the most important preparation for algebra

- Just nine: foundation for arithmetic
- Exact same properties work for whole numbers, fractions, negative numbers, rational numbers, letters, expressions.
- Same properties in 3rd grade and in calculus
- Not just learning them, but learning to use them

Using the properties

- To express yourself mathematically (formulate mathematical expressions that mean what you want them to mean)
- To change the form of an expression so it is easier to make sense of it
- To solve problems
- To justify and prove

Properties are like rules, but also like rights

- You are allowed to use them whenever you want, never wrong.
- You are allowed to use them in any order
- Use them with a mathematical purpose

Properties of addition

Associative property of addition	$(a + b) + c = a + (b + c)$ $(2 + 3) + 4 = 2 + (3 + 4)$
Commutative property of addition	$a + b = b + a$ $2 + 3 = 3 + 2$
Additive identity property of 0	$a + 0 = 0 + a = a$ $3 + 0 = 0 + 3 = 3$
Existence of additive inverses	For every a there exists $–a$ so that $a + (–a) = (–a) + a = 0$. $2 +(-2) = (-2) + 2 = 0$

Properties of multiplication

Associative property of multiplication	$(a \times b) \times c = a \times (b \times c)$ (2 x 3) x 4 = 2 x (3 x 4)
Commutative property of multiplication	$a \times b = b \times a$ *2 x 3 = 3 x 2*
Multiplicative identity property of 1	$a \times 1 = 1 \times a = a$ *3 x 1 = 1 x 3 = 3*
Existence of multiplicative inverses	For every $a \neq 0$ there exists $1/a$ so that $a \times 1/a = 1/a \times a = 1$ 2 x 1/2 = 1/2 x 2 = 1

Linking multiplication and addition: the ninth property

- *Distributive property of multiplication over addition*

$$a \times (b + c) = (a \times b) + (a \times c)$$

$$a(b+c) = ab + ac$$

Find the properties in the multiplication table

- There are many patterns in the multiplication table, most of them are consequences of the properties of operations:
- Find patterns and explain how they come from the properties.
- Find the distributive property patterns

Grade level examples

- 3 packs of soap
- 4 dealing cards
- 5 sharing
- 6 money
- 7 lengths (fractions)
- 8 times larger (%)

“Properties of Operations”

- Also called “rules of arithmetic” , “number properties”

Subtraction and division

- Notice that a + (-a) = 0 let’s define subtraction: a - b = a + (-b)
- And a x (1/a) = 1 let’s us define division:

  a ÷ b = a x (1/b)

Subtraction does not commute. Why not?

That is , does a - b = b - a? Why not?

Use the definition of subtraction and the properties to justify your answer.

Does division commute?

When in doubt, go back to the defintions.

Take the number apart?

Tina, Emma, and Jen discuss this expression:

- 5 *1/3* x 6
- Tina: I know a way to multiply with a mixed number, like 5 *1/3* , that is different from the one we learned in class. I call my way "take the number apart." I'll show you.

Which of the three girls do you think is right? Justify your answer mathematically.

First, I multiply the 5 by the 6 and get 30.

Then I multiply the 1/3 by the 6 and get 2. Finally, I add the 30 and the 2, which is 32.

- Tina: It works whenever I have to multiply a mixed number by a whole number.
- Emma: Sorry Tina, but that answer is wrong!
- Jen: No, Tina's answer is right for this one problem, but "take the number apart" doesn't work for other fraction problems.

What is an explanation?

Why you think it’s true and why you think it makes sense.

Saying “distributive property” isn’t enough, you have to show how the distributive property applies to the problem.

Example explanation

Why does 5 1/3 x 6 = (6x5) + (6x1/3) ?

Because

5 1/3 = 5 + 1/3

6(5 1/3) =

6(5 + 1/3) =

(6x5) + (6x1/3) because a(b + c) = ab + ac

Mental math

72 -29 = ?

In your head.

Composing and decomposing

Partial products

Place value in base 10

Factor $X^2 + 4x + 4$ in your head

Units are things you count

- Objects
- Groups of objects
- 1
- 10
- 100
- ¼   unit fractions
- Numbers represented as expressions

Units add up

- 3 pennies + 5 pennies = 8 pennies
- 3 ones + 5 ones = 8 ones
- 3 tens + 5 tens = 8 tens
- 3 inches + 5 inches = 8 inches
- 3 ¼ inches + 5 ¼ inches = 8 ¼ inches
- ¾ + 5/4 = 8/4
- 3(x + 1) + 5(x+1) = 8(x+1)

Operations and the problems they solve

- Tables 1 and 2 on pages 88 and 89

From table 2 page 89

- *$a \times b = ?$*
- *$a \times ? = p$, and $p \div a = ?$*
- *$? \times b = p$, and $p \div b = ?$*

- *1.Play with these using whole numbers,*
- *2.make up a problem for each.*
- *3. substitute $(x – 1)$ for b*

Write 3 word problems for $y = rx$, where r is a rate.

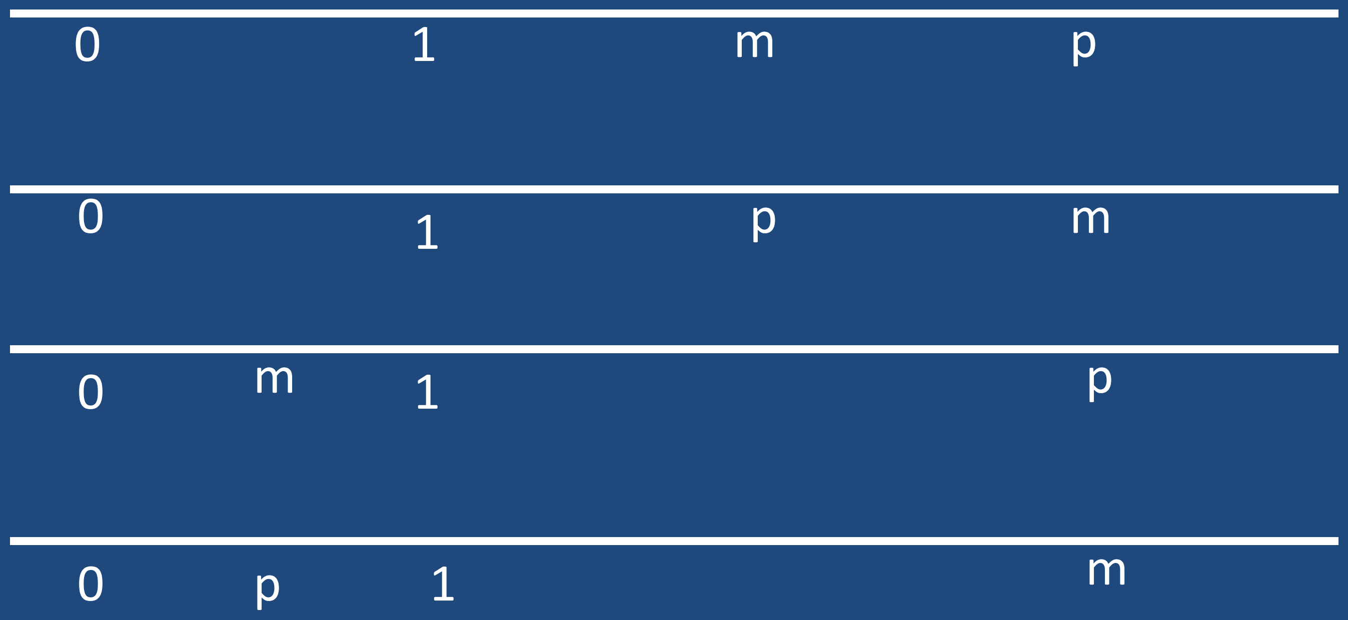
a) When r and x are given

b) When y and x are given

c) When y and r are given

Locate the difference, p - m, on the number line:

For each of the following cases, locate the quotient *p/m* on the number line :



Differentiated Lessons

Differentiating lesson by lesson

Differentiating lesson by lesson:

- The arc of the lesson
- The structure of the lesson
- Using a problem to teach mathematics
- Classroom management and motivation
- Student thinking and closure

The arc of the lesson

- Diagnostic: make differences visible; what are the differences in mathematics that different students bring to the problem
- All understand the thinking of each: from least to most mathematically mature
- Converge on grade -level mathematics: pull students together through the differences in their thinking

Next lesson

- Start all over again
- Each day brings its differences, they never go away

Lesson Design

- Problem of the Day
- Lesson Opener
- Comprehensible Input/Modeling and Structured Practice
- Guided Practice
- Presentation (by student)
- Closure
- Preview

This design works well for introducing new procedural content to a group within range of the content

Adapted Lesson Structure

Adapted Lesson Structure for differentiating

• Pose problem	whole class	(3-5 min)
• Start work	solo	(1 min)
• Solve problem	pair	(10 min)
• Prepare to present	pair	(5 min)
• Selected S presents	whole cls	(15 min)
• Closure & Preview	whole cls	(5 min)

Posing the problem

- Whole class: pose problem, make sure students understand the language, no hints at solution
- Focus students on the problem situation, not the question/answer game. Hide question and ask them to formulate questions that make situation into a word problem
- Ask 3-6 questions about the same problem situation; ramp questions up toward key mathematics that transfers to other problems

What problem to use?

- Problems that draw thinking toward the mathematics you want to teach. NOT too routine, right after learning how to solve
- Ask about a chapter: what is the most important mathematics students should take with them? Find a problem that draws attention to this mathematics
- Begin chapter with this problem (from lesson 5 thru 10, or chapter test). This has diagnostic power. Also shows you where time has to go.
- Also Near end of chapter, while still time to respond

Solo-pair work

- Solo honors ‘thinking’ which is solo
- 1 minute is manageable for all, 2 minutes creates classroom management issues that aren’t worth it.
- An unfinished problem has more mind on it than a solved problem
- Pairs maximize accountability: no place to hide
- Pairs optimize eartime: everyone is listened to
- You want divergance; diagnostic; make differences visible

Presentations

- All pairs prepare presentation
- Select 3-5 that show the spread, the differences in approaches from least to most mature
- Interact with presenters, engage whole class in questions
- Object and focus is for all to understand thinking of each, including approaches that didn't work; slow presenters down to make thinking explicit
- Go from least to most mature, draw with marker correspondences across approaches
- Converge on mathematical target of lesson

Close

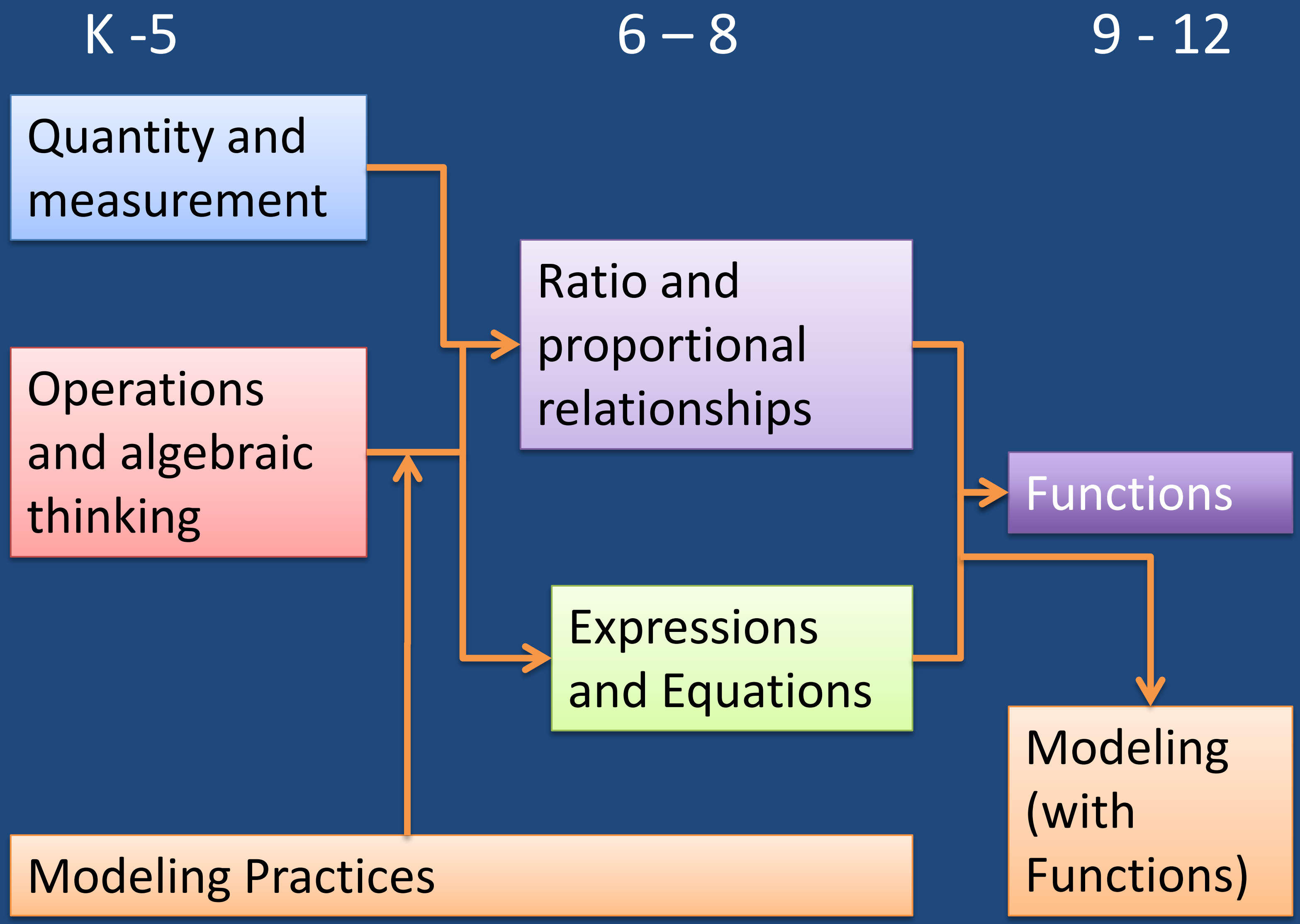
- Use student presentations to illustrate and explain the key mathematical ideas of lesson
- Applaud
  - adaptive problem solving techniques that come up,
  - the dispositional behaviors you value,
  - the success in understanding each others thinking (name the thought)

The arc of a unit

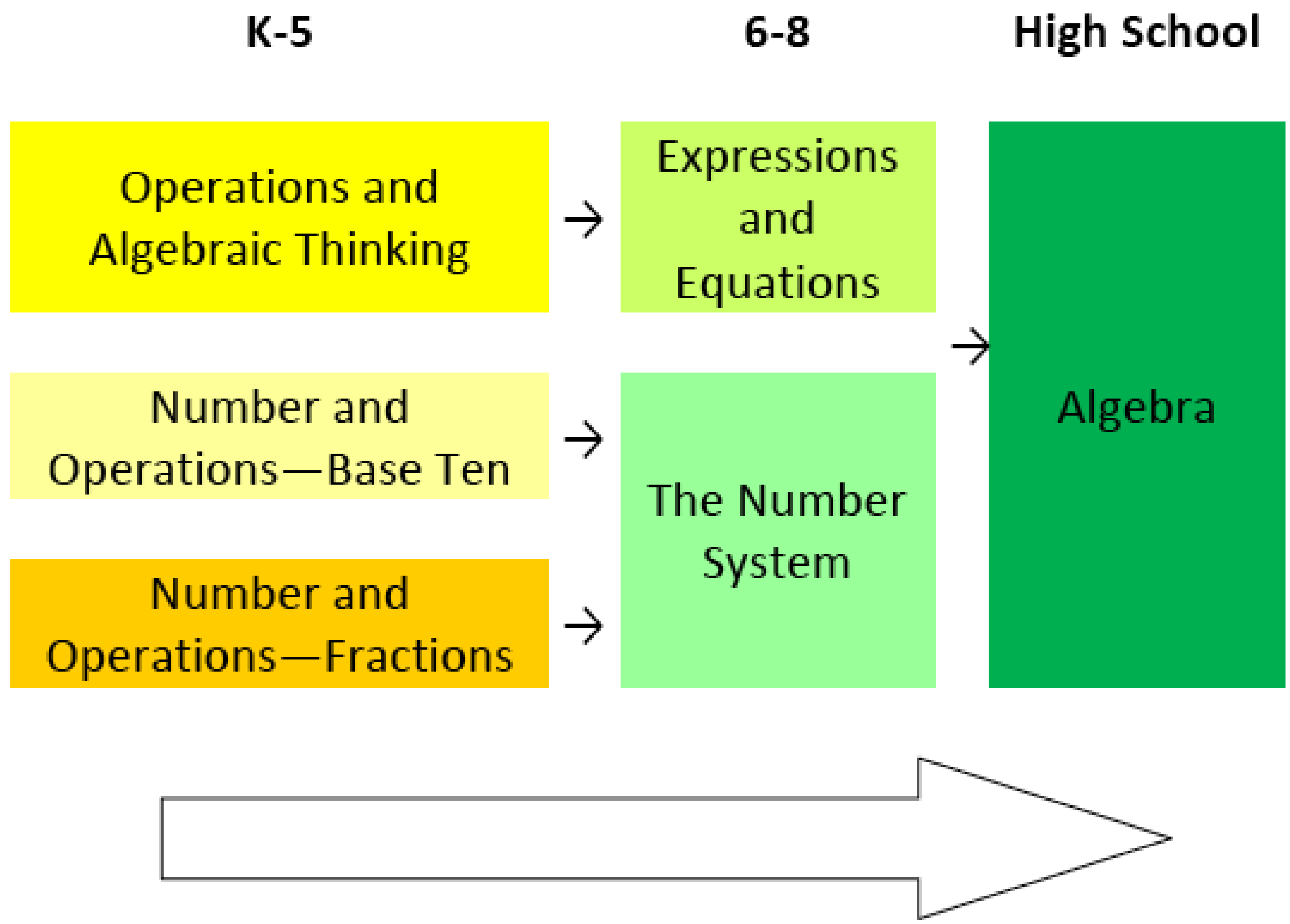
- Early: diagnostic, organize to make differences visible
  - Pair like students to maximize differences between pairs
- Middle: spend time where diagnostic lessons show needs.
- Late: converge on target mathematics
  - Pair strong with weak students to minimize differences, maximize tutoring

Each lesson teaches the whole chapter

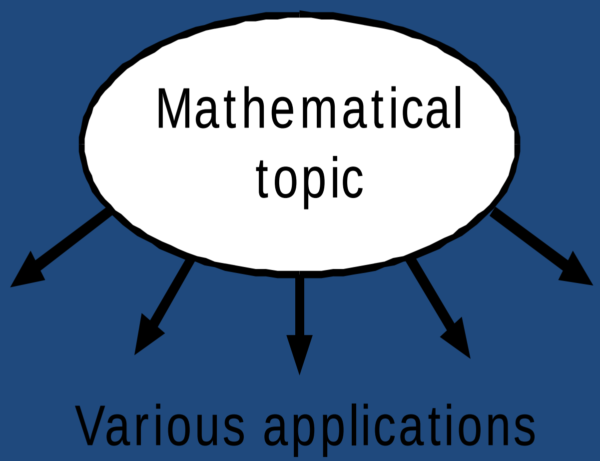
- Each lesson covers 3-4 weeks in 1-2 days
- Lessons build content by
  - increasing the resolution of details
  - Developing additional technical know-how
  - Generalizing range and complexity of problem situations
  - Fitting content into student reasoning
- This is not “spiraling”, this is depth and thoroughness for durable learning

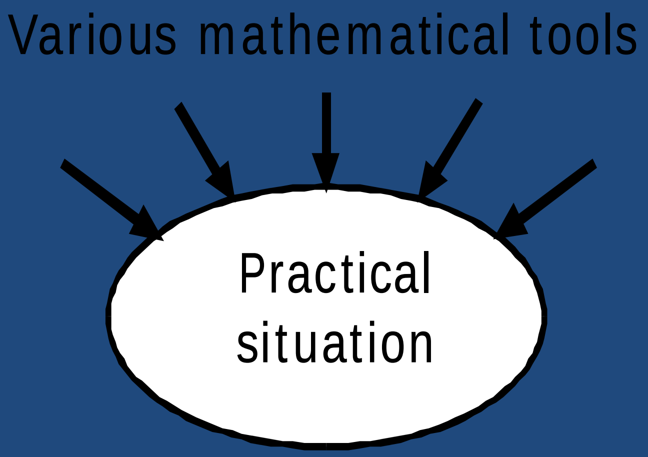

K -5
6 – 8
9 - 12
Quantity and measurement
Operations and algebraic thinking
Ratio and proportional relationships
Expressions and Equations
Functions
Modeling (with Functions)
Modeling Practices

Focusing attention within Number and Operations



“Concept focused” v “Problem focused”:





Summative and Formative Assessments

Next Steps

Promises to Keep

- Promise to the student:
  - "Study and learn what is in this syllabus, do your assignments and we promise you will do well on the test."
- Charge to the examination system:
  - "Design and use an examination that keeps that promise."

Validity of State Test

- Validity is a property of a use of an assessment, not of the assessment per se.
- The intended use of state tests is to motivate and steer the schools with carrots and sticks based on the test.
- Are the tests valid for this use?
- Empirical validity question: are districts, schools, teachers, students motivated and steered in the right directions?
- hmmm

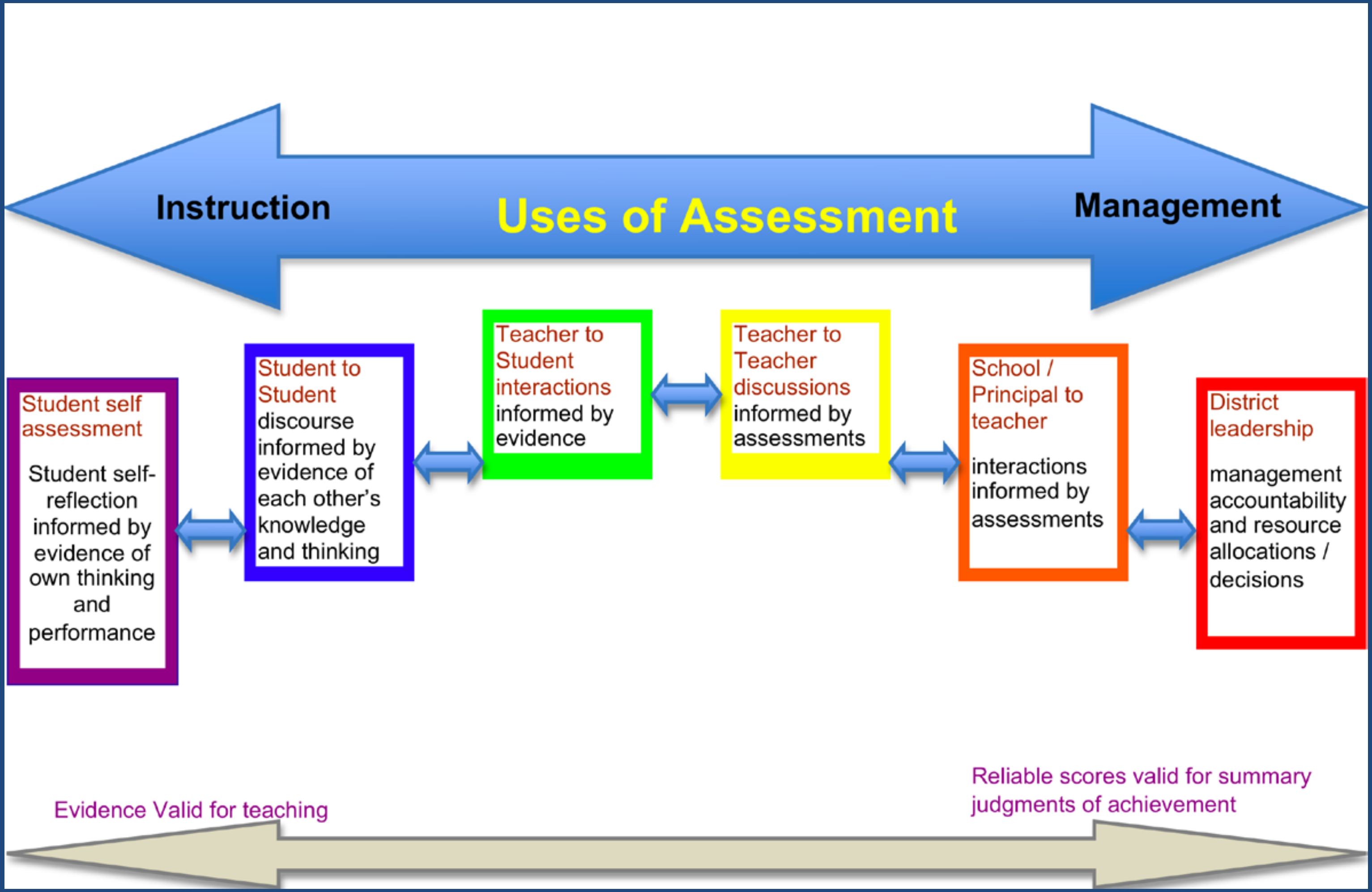
Improve validity

- Validity of using of formative and summative assessments for steering the system at each level of the system
- Too many “periodic” assessments at the district level are images of the state test which is a poor image of the curriculum described in the standards. State tests were not designed with this use in mind.
- Design model formative assessments first and then make state tests in their image.

What makes an assessment formative?

- Its use to inform instruction.
- Timing: assessment available and used while instruction is still going on, while there is still time for instruction to respond to information.
- Feedback: informs, that is the feedback has content (mathematical content) not just value judgment
- Motivates: assessment promises to respond to growth; relationship between test items and what should be studied is transparent and direct.
- Test items look like the classwork and homework implied by standards; not like a psychological instrument.

The Last Mile: Assessment Purposes that reach the student



Grades

- Feedback vs. grades
  - Need to separate feedback from grading
- At least half the course grade should be based on revised work
  - Revision based on feedback
- Accomplishment vs. differences

Variability among students

- Variability (differences) is essential for scaling in measurement, but not he foundation for feedback
  - Indeed, it is a standard definition of “information”…information means how well the measure discriminates among students

Accomplishment means

- a standard has been set that is worth achieving,
- criteria for judging successful achievement are transparent,
- Students can try, try again
- Students are supposed to seek and get help

Should be at least half the grade

Issues for Summative Tests

- Fit to standards: hits the priorities
- Fit to population: detects growth across the distribution
- Designed to detect the growth along progressions in the standards (progressions are not the same as “difficulty”)
- Constraints: time and $ and timing
- Worthy of imitation for local periodic assessments: shows thinking, knowledge

Trade -offs

- Optimizing score vs.. optimizing information about student knowledge, reasoning and how they get the wrong (or right) answer.
- Optimizing aggregating reports up vs. information inside the opportunity to teach.
- Need both implies need to allocate time and resources to both.

Duplicity of Purposes: Out

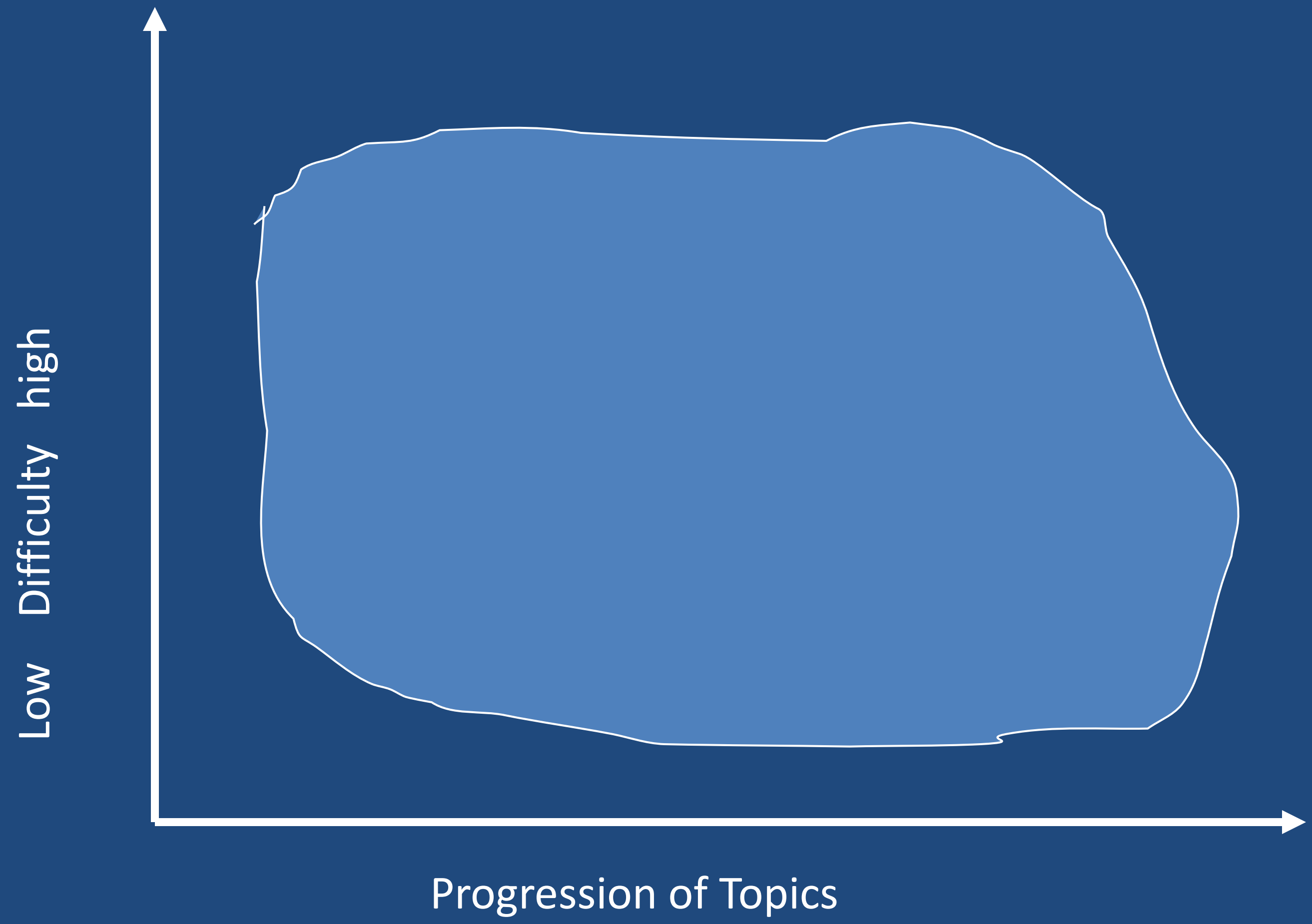
- Out facing reports require summary and aggregation. To add up, we need common units: inches to inches, dust to dust. If we have apples and oranges, we need a common denominator: 3 apples + 5 oranges = 8 fruit. Adding up requires blurring distinctions.
- Reliablity of scores is a measure of unidimensionality: apples to apples; average correlation of items to the total score adjusted to sample size (number of items).
- Assessments that optimize reliable scores have small value for the kinds instructional choices teachers make; encourage seeing intelligence as fixed.

Duplicity of Purposes: In

- To improve a students spelling, the teacher needs to see the misspellings and correct them, perhaps teach spelling heuristics that fit student's patterns. Score has small formative value compared actual spellings.
- Knowing that a student scores low on fractions has some small value to a teacher (scores low on mathematics has even less value).
- What a teacher needs is to see: the student working fraction problems, see where he goes wrong and give feedback that corrects what student actually does.

Tasks by progression and depth of practice

Tasks by progression and difficulty (p)



Tasks by depth of practice and difficulty

Uptake and usability

- Design for evolution in a community of practice. Provide infrastructure and support for community of practice.
- Specify the baseline stuff. Standards should list the ingredients needed (kits, crayons, books, technology,) ; cannot require paper and pencil algorithms without supplying paper and pencil.
- Teach less, learn more.

Design for Evolution of practice rather than alignment of documents

- Legalistic standards say: lead and manage your system in alignment with these lists. Leads to legalistic tools and relationships, managerial compliance approach, cover these and check them off . distracts from teaching and learning, lacks focus.

Standards should be alive, not published to death.

- When standards are issued, issue the infrastructure for practitioner community to blog, share video, lessons, problems. The infrastructure should not be hierarchically controlled but belong to its users. Community. Wiki.
  Organize Tips from the trenches includes lessons, teacher blog, video clips, problems.

Inclusion, equity and social justice

- Standards should be within reach of the distribution of students.
- Focus so that there is time to be patient.
- Understanding thinking of others should be part of the standards, using the discipline's forms of discourse
- Pathways for students includes way for children to catch up.
- Standards that require less than the available time – teach less, learn more

The user has control

- Sometimes a tool is just right for the wrong use.

4 SPEED

Distance, time and speed

Include:

- concepts of speed and average speed,
- relationship between distance, time and speed
  - Distance = Speed × Time,
  - Speed = Distance ÷ Time,
  - Time = Distance ÷ Speed,
- calculation of speed, distance or time given the other two quantities,
- writing speed in different units such as km/h, m/min, m/s and cm/s,
- solving up to 3-step word problems involving speed and average speed.

Exclude conversion of units, e.g. km/h to m/min. 30m/h

One of 8

2.0 Students analyze and use tables, graphs, and rules to solve problems involving rates and proportions:

2.1 Convert one unit of measurement to another (e.g., from feet to miles, from centimeters to inches).

2.2 Demonstrate an understanding that rate is a measure of one quantity per unit value of another quantity.

2.3 Solve problems involving rates, average speed, distance, and time.