The Literacy Gap

Identifying cognitive processes important to mathematics learning but often overlooked

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Overview

- Identifying the problem
  - Performance on PISA test items
- Drivers of mathematical literacy
  - PISA research outcomes
  - The ‘fundamental mathematical capabilities’
- Action required
  - Implications for mathematics classrooms

The *mathematics terror* index

terror

Ongoing torment,
cold sweats

avoidance

Long forgotten,
I make do

comfort

Confident with
day to day
challenges and
opportunities

expertise

Advanced
technical skills,
professional use

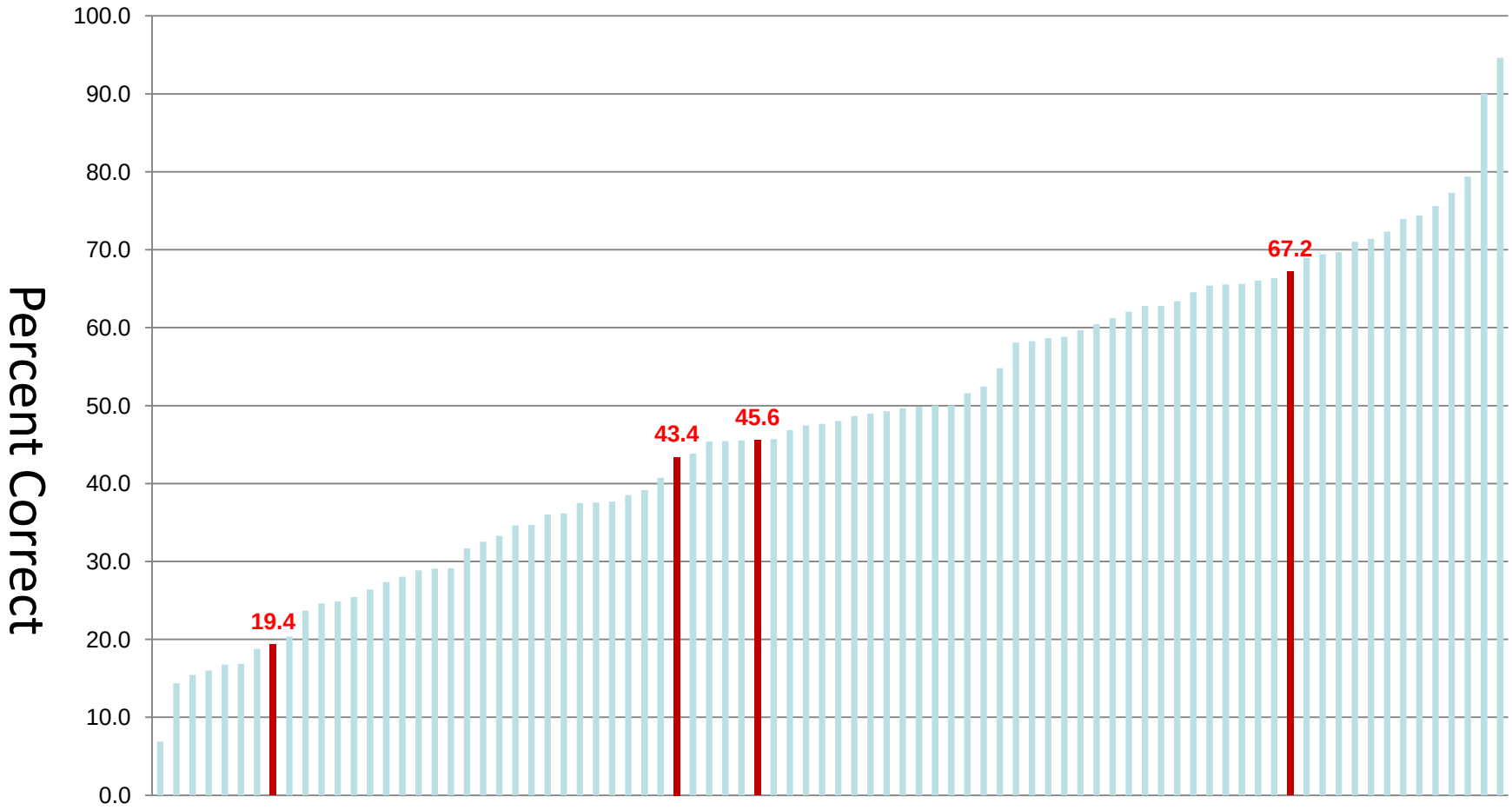
Some PISA test items

- Publicly released items (50 units, 90 Qs)
  - https://mypisa.acer.edu.au/

    >> Publications >> Instruments >>

    "Released PISA Items – Maths – Prior PISA2006
  - "Take The Test" (OECD, 2009)
- Four items from 2003
  - Exports Q1, Q2
  - Litter
  - Carpenter

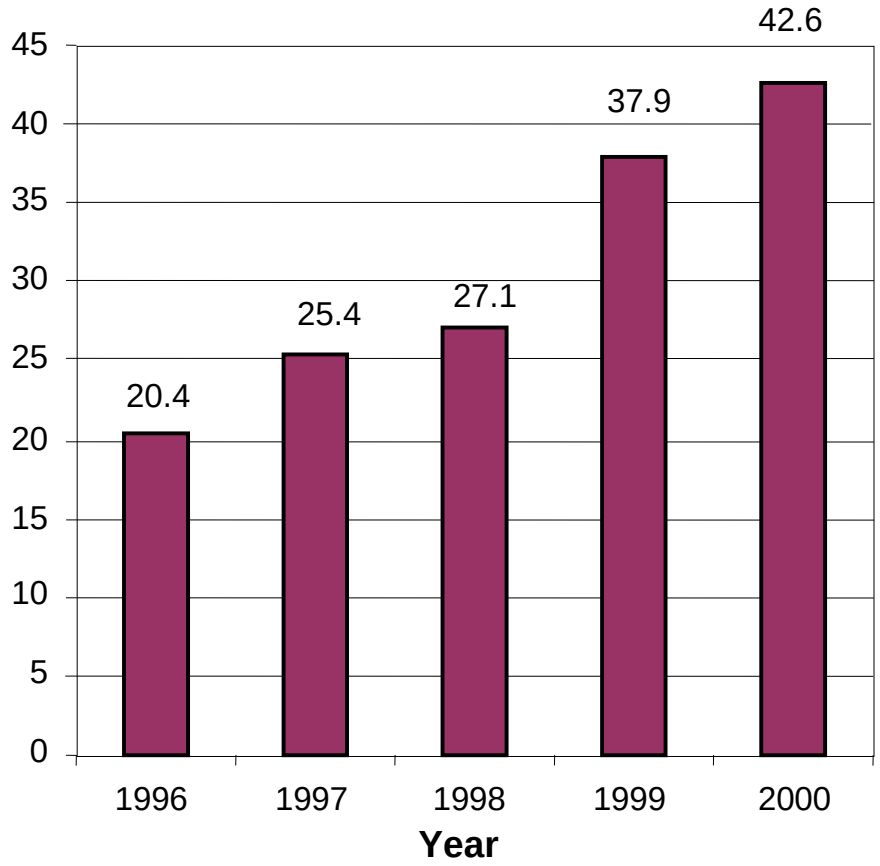
Difficulty of items (2003)



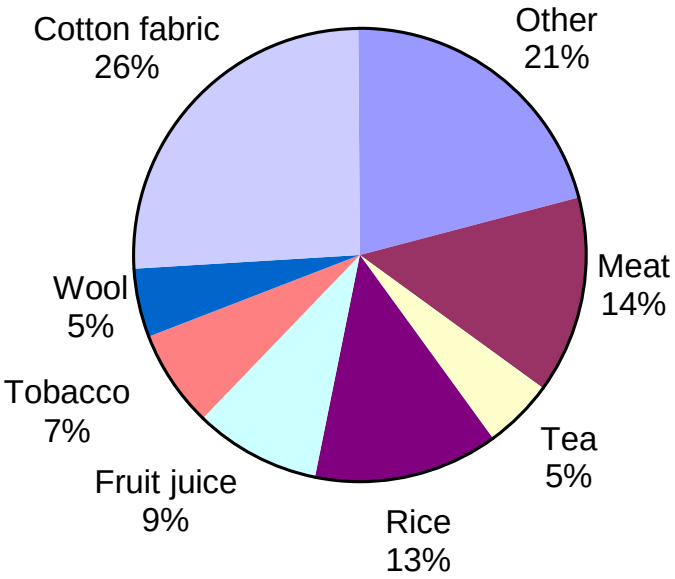
EXPORTS

The graphics below show information about exports from Zedland, a country that uses zeds as its currency.

Total annual exports from Zedland in millions of zeds, 1996-2000



Distribution of exports from Zedland in 2000



Q 1: What was the total value (in millions of zeds) of exports from Zedland in 1998?

Q 2: What was the value of fruit juice exported from Zedland in 2000?

A 1.8 million zeds. **B** 2.3 million zeds. **C** 2.4 million zeds. **D** 3.4 million zeds. **E** 3.8 million zeds.

LITTER

For a homework assignment on the environment, students collected information on the decomposition time of several types of litter that people throw away:

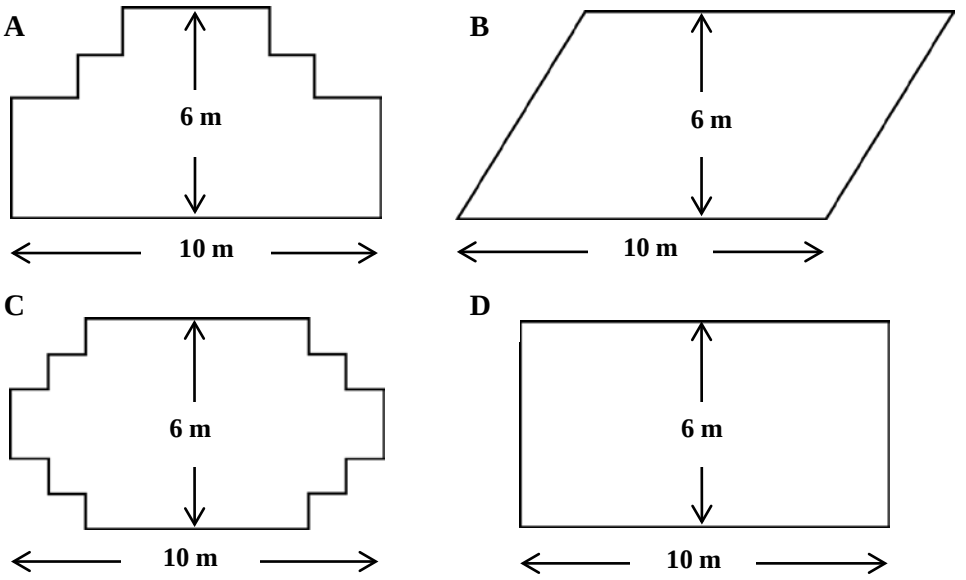
Type of Litter	Decomposition time
Banana peel	1–3 years
Orange peel	1–3 years
Cardboard boxes	0.5 year
Chewing gum	20–25 years
Newspapers	A few days
Polystyrene cups	Over 100 years

A student thinks of displaying the results in a bar graph.

Give **one** reason why a bar graph is unsuitable for displaying these data.

CARPENTER

A carpenter has 32 metres of timber and wants to make a border around a garden bed. He is considering the following designs for the garden bed.



Circle either "Yes" or "No" for each design to indicate whether the garden bed can be made with 32 metres of timber.

Garden bed design	Using this design, can the garden bed be made with 32 metres of timber?
Design A	Yes / No
Design B	Yes / No
Design C	Yes / No
Design D	Yes / No

PISA Results

Question	Facility (all students)	Facility (AUS students)
Exports Q1	67.2%	85.8%
Exports Q2	45.6%	46.3%
Litter	43.4%	64.1%
Carpenter	19.4%	23.3%

Is it good enough?

- Explanations? No!
  - Lack of required mathematical knowledge?
  - Inability to ACTIVATE relevant mathematical knowledge?
- How do we meet opportunities to use mathematics?

Fundamental mathematical capabilities

- Seven capabilities that
  - Underpin the PISA mathematics framework, and
  - Explain up to 70% of the variability in item difficulty

Fundamental mathematical capabilities

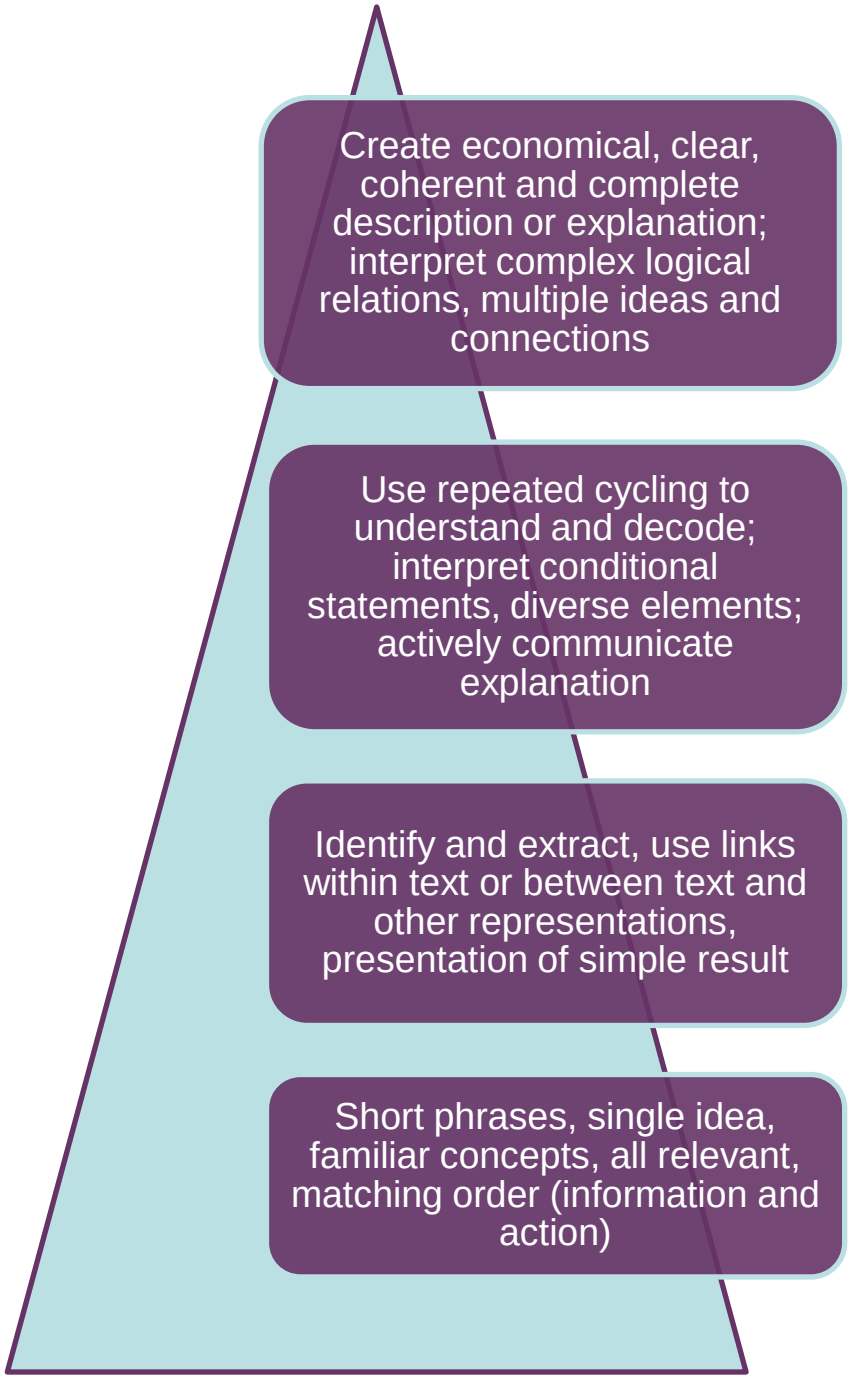
- Communication
- Mathematising
- Representation
- Reasoning and argument
- Strategic thinking
- Using symbolic, formal and technical language and operations
- [Using mathematical tools]

Item difficulty research

- Six capabilities identified
- Each capability defined, and described at four levels
- Process devised for rating of items by expert raters
- Analysis of rating data (consistency among raters, regression used to link ratings to empirical item difficulty)
- Variation explained – about 70%

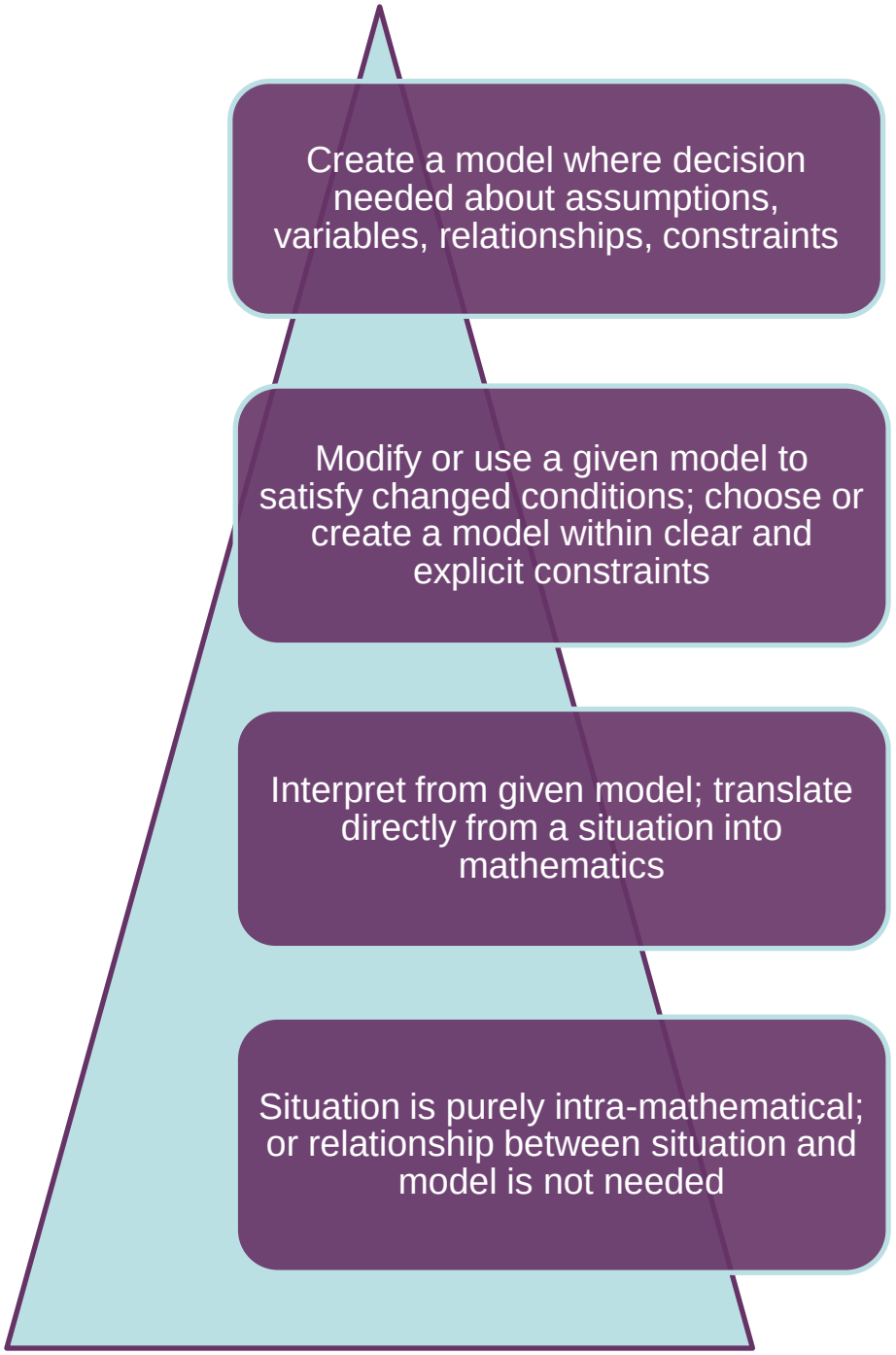
Communication

- Incoming
  - Reading, decoding, interpreting statements and mathematical information
    - Very common expectation
- Outgoing
  - Explaining, presenting, arguing
    - Not so common, and very difficult for many students



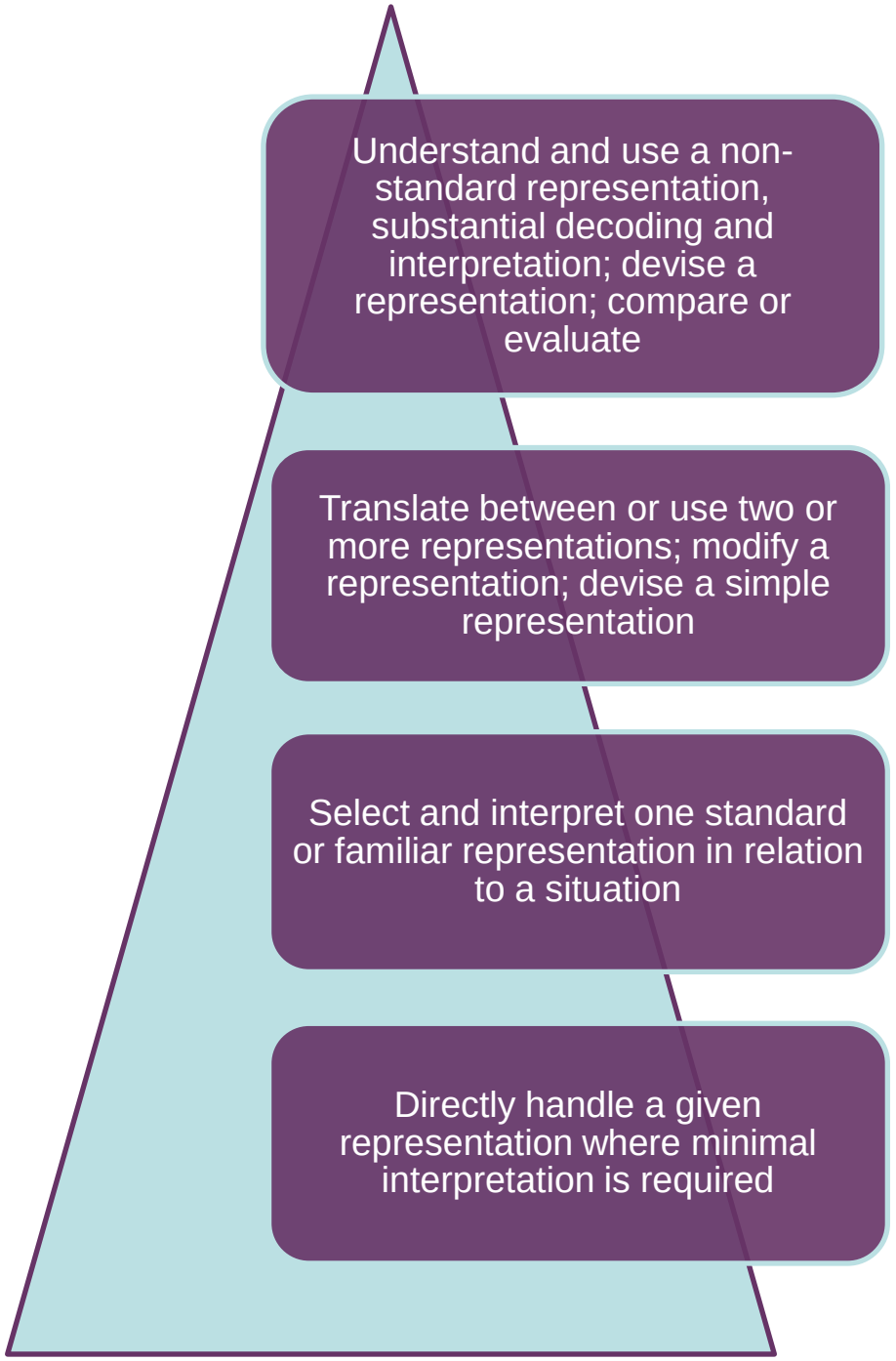
Mathematising

- Transform a real world problem into a mathematical problem
  - Structuring, idealising, making assumptions, building a model
- Interpret mathematical objects or information in relation to the situation represented
  - evaluate or validate a solution in relation to the original problem



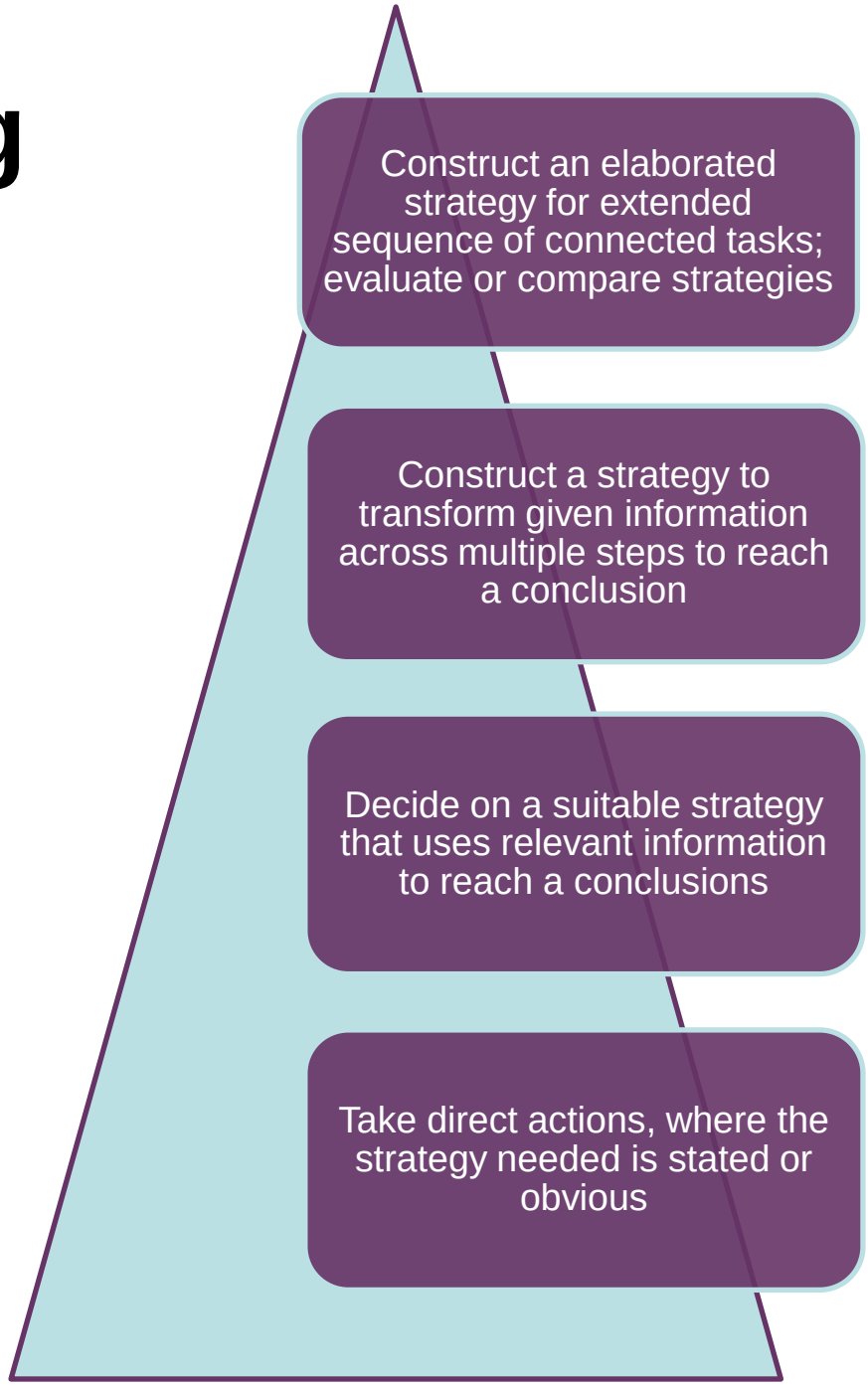
Representation

- Devising or using depictions of mathematical objects or relationships: equations, formulae, graphs, tables, diagrams, textual descriptions, …
  - Interpreting, translation between, and making use of given representations
  - Selecting or devising representations to capture a situation or to present one's work



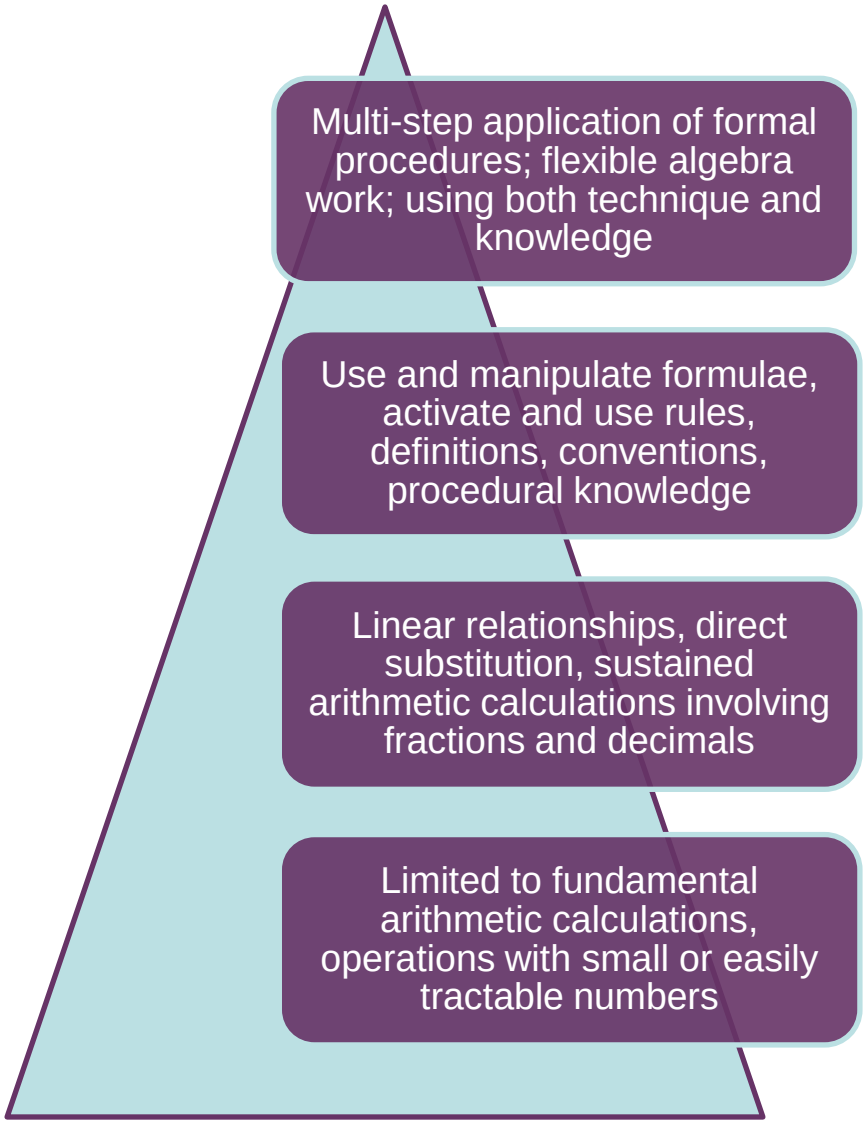
Strategic thinking

- Selecting or devising, and implementing, a mathematical strategy to solve problems arising from the task or context
  - Metacognitive control process



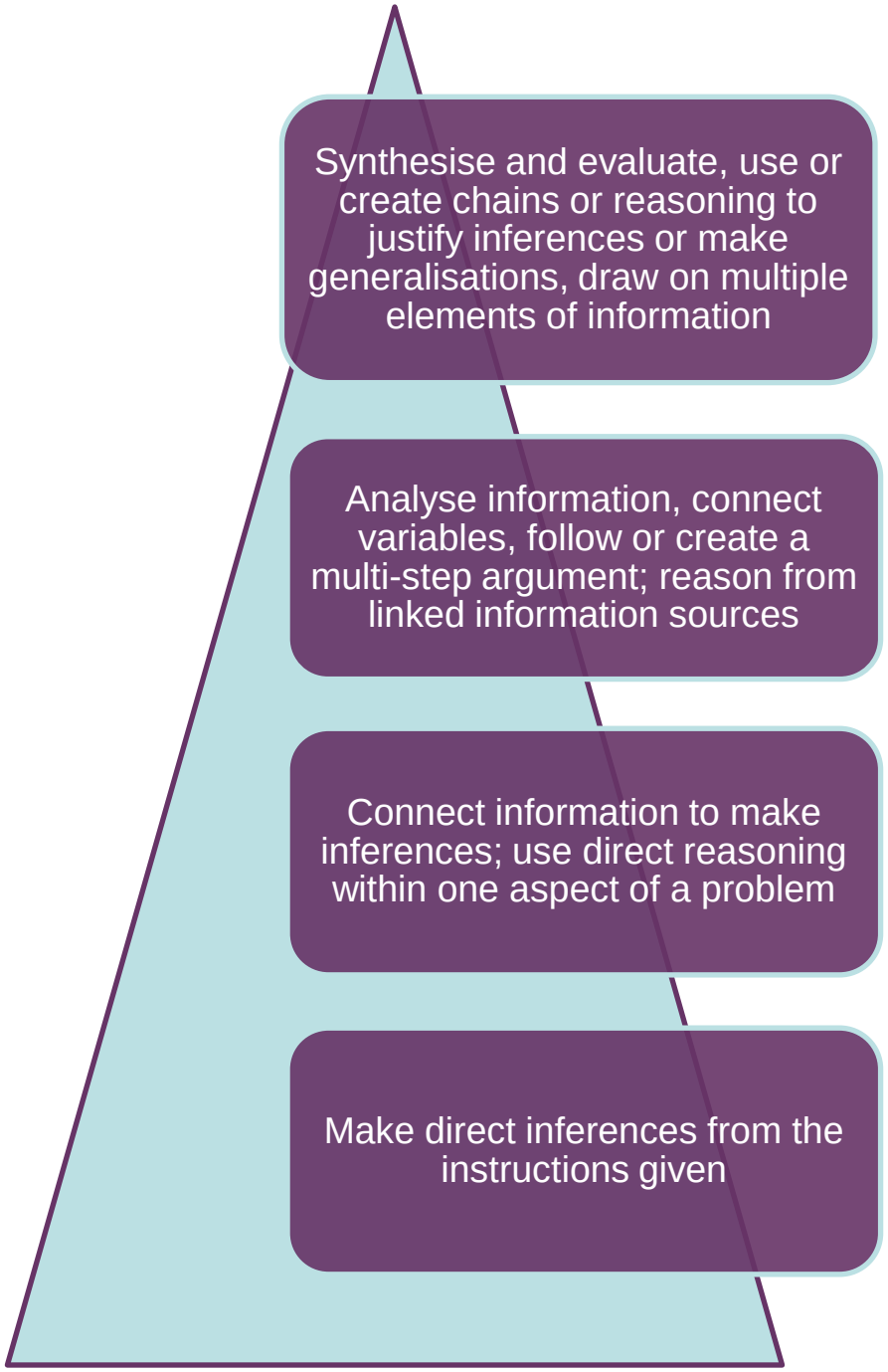
Using symbolic, formal and technical language and operations

- Understanding, manipulating, and making use of symbolic expressions; using constructs based on definitions, rules and conventions, formal systems



Reasoning and argument

- Logically rooted thought processes that explore and link problem elements to make inferences from them; or to check a given justification; or to provide a justification



Ratings for ‘Exports Q1’

Category	Description	Ratings
Communication	Read and interpret text and two graphs, identify and present required information	1, 1, 2 (mean=1.3)
Representation	Link text and graphic, locate data from correct graph	1, 1, 1 (mean=1.0)
Mathematising	Already mathematised, link to situation only minimal	1, 0, 0 (mean=0.3)
Strategic thinking	Obvious strategy, direct action (locate and extract the required value)	0, 0, 0 (mean=0.0)
Symbols and formalism	Read required value from graph, use correct units	0, 1, 0 (mean=0.3)
Reasoning and argument	Follow direct instructions	0, 1, 0 (mean=0.3)

Ratings for ‘Exports Q2’

Category	Description	Ratings
Communication	Read and interpret text and two graphs, identify and select required information	1, 1, 2 (mean=1.3)
Representation	Link text and two graphs, locate related data correctly from each graph	2, 2, 2 (mean=2.0)
Mathematising	Find a mathematical way to express ‘value of fruit juice’	1, 0, 1 (mean=0.7)
Strategic thinking	Connect the two graphs, extract data and perform calculation (multi-step process)	2, 0, 1 (mean=1.0)
Symbols and formalism	Read required values from each graph, perform the correct calculation	0, 1, 1 (mean=0.7)
Reasoning and argument	Follow chain of reasoning to connect the two graphs, extract and process data	1, 1, 1 (mean=1.0)

Ratings for ‘Litter’

Category	Description	Ratings
Communication	Read and interpret text and table; construct a written argument	2, 2, 1 (mean=2.6)
Representation	Link text and related graphs (either imagined or sketched)	3, 2, 2 (mean=2.3)
Mathematising	Identify mathematical characteristics of bar graphs for each litter type	1, 0, 1 (mean=0.7)
Strategic thinking	Strategy is stated in question	0, 0, 0 (mean=0.0)
Symbols and formalism	Knowledge of bar graph construction, understanding scale	0, 1, 0 (mean=0.3)
Reasoning and argument	Formulate argument in mathematical terms	1, 2, 1 (mean=1.3)

Ratings for ‘Carpenter’

Category	Description	Ratings
Communication	Read and interpret text and construct a written argument	2, 2, 1 (mean=2.6)
Representation	Link text and related diagrams	1, 1, 1 (mean=1.0)
Mathematising	Recognise relevance of total height and width in relation to perimeter	1, 0, 1 (mean=0.7)
Strategic thinking	Strategy to calculate or compare perimeters with only partial information	2, 1, 1 (mean=1.3)
Symbols and formalism	Process and analyse dimensions, simple addition	1, 1, 1 (mean=1.0)
Reasoning and argument	Follow sustained chain of reasoning to analyse each of four shapes	2, 3, 2 (mean=2.3)

PISA Results - reprise

Question	Facility (all students)	Rating totals
Exports Q1	67.2%	3.2
Exports Q2	45.6%	6.7
Litter	43.4%	7.2
Carpenter	19.4%	8.9

The message?

- Possession of these capabilities is crucial to the activation of one's mathematical knowledge
- These capabilities drive the use of our mathematical knowledge to solve contextualised problems
- We can't afford to ignore the importance of these capabilities

The action required?

- Attention must be paid to fostering these capabilities in maths classrooms
- How? Consider …
  - The nature of classroom discussion
  - Opportunities/demands for expression, both written and oral
  - The kinds of problems used
- [Also monitor any potential conflicts with other priorities]