THE SOCIAL OUTCOMES OF LEARNING MATHS: STANDARD, UNINTENDED OR VISIONARY?

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Why teach or learn maths?

What are the intended aims and outcomes of school maths?

I distinguish three groups of aims & outcomes:

1. **Standard aims** or basic reasons for teaching maths
2. **Unintended / unexpected outcomes** of maths - for some students or maybe for all?
3. **Visionary aims**
   - What do we wish to see in School Maths?
   - What new emphases would enhance students and society?

STANDARD SCHOOL AIMS

1. **Basic aim for school maths - Functional numeracy.** This is:

- Ability with maths and numeracy skills adequate for successful general employment and functioning in society.
- A basic minimal requirement for all at the end of schooling (excluding few with preventative disability).
- Mostly achieved by end of primary school

2. Practical, work-related knowledge

- Capability to solve practical problems with maths, especially industry and work-centred problems
- Not necessary for all, for the depth and type of problems vary across employment types
- Most occupations requiring maths also provide specialist training
- But a **basis for learning** further specialist knowledge and skills is also needed

3. Advanced specialist knowledge

- Not necessary knowledge for all adults
- Such knowledge needed by minority as foundation for a broad range of university studies, e.g., STEM, medical and social science studies.
- A necessary option in advanced technological society, and more students need encouragement
- Advanced study leads to a highly numerate professional class (e.g., as in France, Hungary)
- **But this aim should not *dominate* or *distort* the maths curriculum for all**

Summary

- **Functional**, **Practical** and **Advanced** knowledge make up useful or necessary maths for some or all
- Primarily for benefit of employment and society from an economic perspective
- Also benefits students in terms of functioning in society, work and further study
- Aim 3 also sustains ***maths itself as a discipline*** – *without specialists maths will not survive*

UNINTENDED OUTCOMES OF SCHOOL MATHEMATICS

- Unintended outcomes include student **values**, **attitudes** and **beliefs** about **maths** developed during years of schooling
- Not planned or intended but outcomes of the **'hidden' curriculum of schooling**
- Includes beliefs about the ***nature of maths***, about ***what is valuable in maths***, and about ***who can be successful in maths***.
- Examples of these beliefs are:

Beliefs about maths include:

- Maths intrinsically **difficult**, inaccessible to most
- Success in maths due to **inherited talent**, not effort
- Maths is a **male domain**, incompatible with femininity
- Maths is **abstract theoretical subject** disconnected from society and day-to-day life
- Maths is **abstract and timeless**, completely **objective** and **absolutely certain**
- Maths is **universal**, **value-free** and **culture-free**.

FALSEHOODS

- Each of these beliefs is **false!**
- Many of my writings devoted to showing this, e.g.

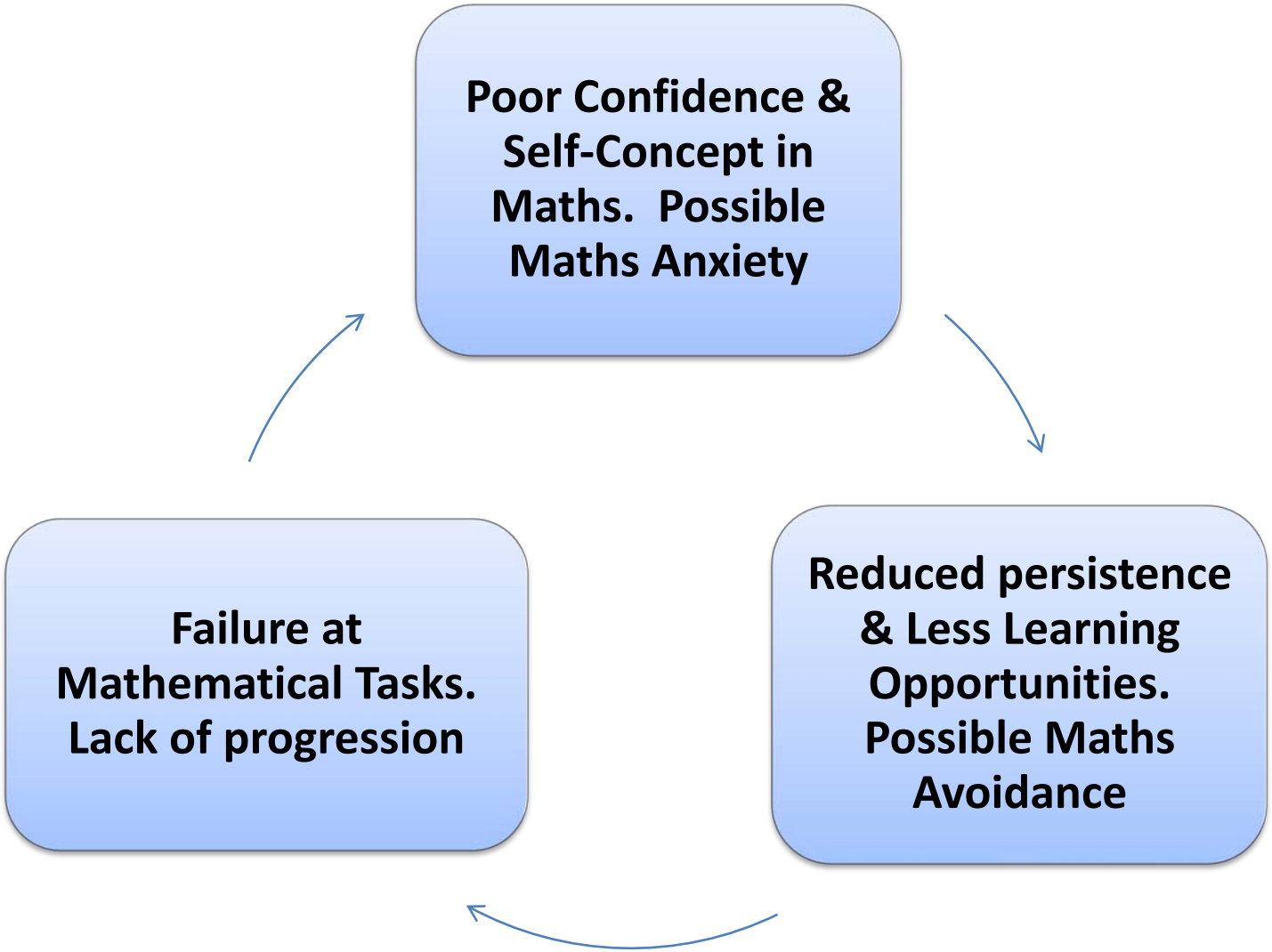
  The Philosophy of Mathematics Education, 1991

  Social Constructivism as a Philosophy of Mathematics, 1998
- Good news is that a growing number of researchers and teachers **reject** these beliefs
- Acceptance varies by culture, e.g., in Far East maths success seen as due to effort not ability.

FALSE BELIEFS

- The bad news is such beliefs are **still held by many** students and parents.
- Such false beliefs are spread
  * via popular *images of maths* in media and society
  * through *image/experience* in some maths classes
- One widespread but not universal outcome - many students develop **negative attitudes** about maths and their own capabilities in maths
- But as **sports** show, good attitudes are vital for success
- Student lack of confidence in own maths ability becomes a self-fulfilling prophecy – **the failure cycle**

The Failure Cycle for Maths



Another False Belief

- Despite progress, mathematics is still widely seen as a **male domain**
- although girls now equal boys in mathematical achievement at 16 years, too many women still
  - * ***doubt*** *their own abilities,* ***lack*** *confidence*
  - * ***choose not to pursue*** *maths related studies or careers*

Beliefs and Inequalities in Maths

Values, images, beliefs & attitudes about maths underlie many differences in learning outcomes across groups defined by

sex
socio-economic status
ethnicity.

E.g., Indigenous mathematics performance in Australia can lag 2+ years behind non-Indigenous students (Queensland Studies Authority, 2004).

But explanation of such inequalities is complex involving social theory, e.g., Bourdieu's cultural capital theory as well as maths beliefs

VISIONARY GOALS FOR MATHS

Move from maths curriculum traditionally **defined as content**. Instead propose **empowering** and **broadening aims** for maths

Students should develop:

4 *Mathematical confidence*
5 *Maths creativity through problem posing and solving*
6 *Critical maths citizenship for social empowerment*
7 *Broader appreciation of mathematics*.

These *not utilitarian aims* - more to do with personal, cultural and social relevance

But have *powerful incidental benefit* for society through there being more empowered and more knowledgeable citizens

4. Maths Attitudes & Confidence

- Attitudes are a **vital** and **underestimated** incidental outcome of school maths
- Attitudes uniquely involve development of whole person in a rounded way: **intellect** *and* **feelings**.
- Effective maths knowledge & capabilities rest on
  - * *Feelings of maths enablement and empowerment*
  - * *Confidence to learn and apply maths*
  - * *Enjoyment in learning and using maths*
  - * *Freedom from negative attitudes to maths*

Mathematical Confidence

Maths confidence includes

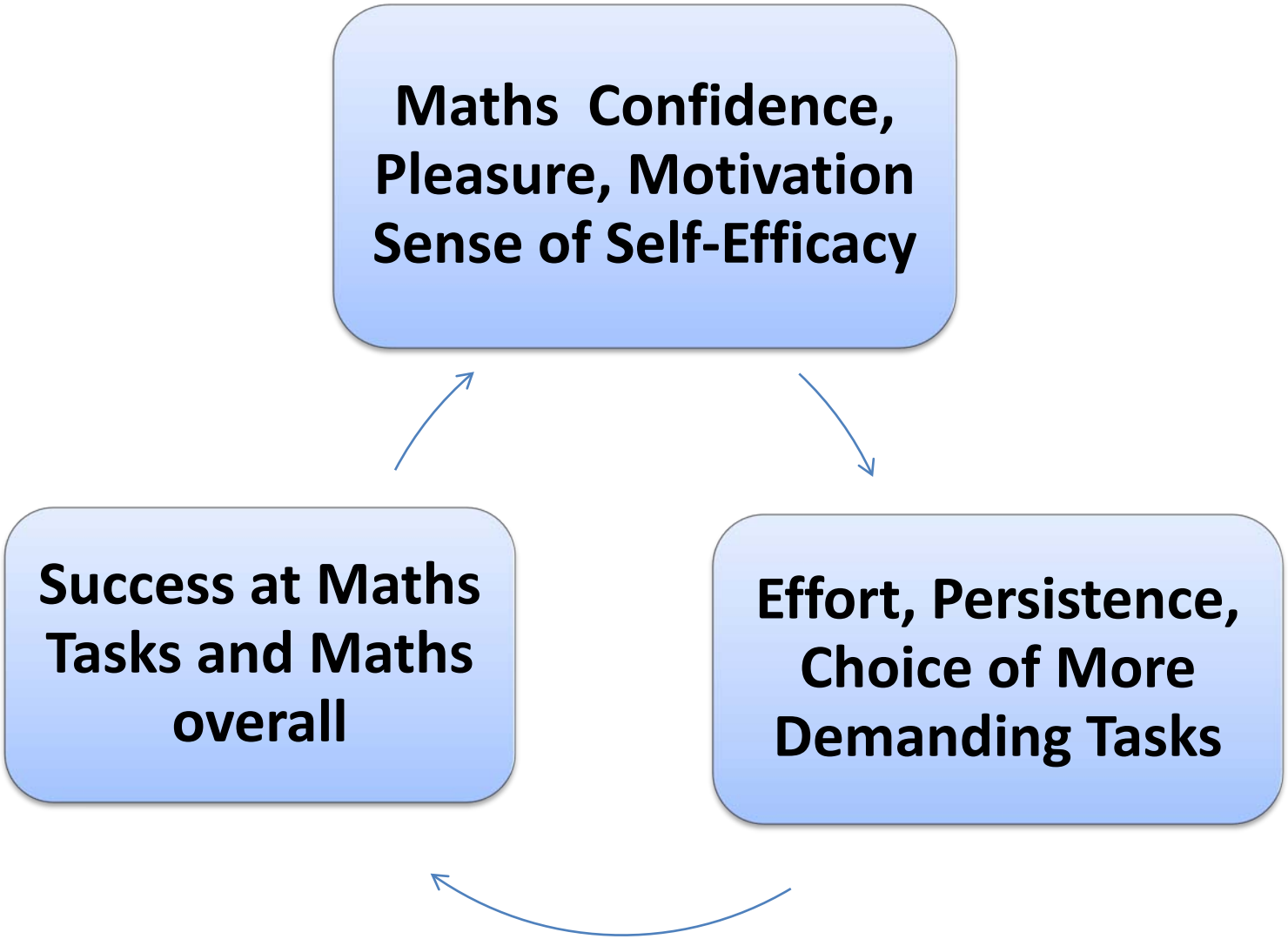
** being confident in one's personal knowledge of maths*
** feeling able to use and apply maths widely*
** confidence in acquiring of new knowledge and skills when needed*

Such confidence leads to

I Persistence in solving difficult mathematical problems,
II Willingness to accept challenging tasks

This is reflected in the virtuous and upwardly spiralling Success Cycle

The Success Cycle in Maths



Building Success

- The success cycle is an intrinsic mechanism drawing us to engagement and effort powered by the pleasures of success and self-enhancement - *like a light draws a moth*.
- We can turn a failure cycle into a success cycle by ***subtracting risk*** and ***making success achievable***
- Done right this should lead to ***positive attitudes***
- In school this means reducing the **dominance of exams** and improving quality of student learning experience (more choice, interest, effort, success)

5. Problem Posing and Solving

- Maths too often seen as **non-creative** and **mechanical**
- Problem solving too often focussed on ***routine*** problems
- Great potential for creativity in school maths, but it lies in solving ***non-routine*** problems
- True problem solving requires new methods and approaches to be created for non-routine solutions
- Students **choose** what models and approaches to use in their solutions
- They **create** and apply heuristics – strategic solution methods

Problem Posing

Problem posing is the formulation of **new maths questions** and **problems** to be solved

Problem posing involves students:

* *Choosing aspects of life to probe, model and explain*
* *Choosing mathematical patterns to investigate*
* *Framing questions by generalization, analogy*, etc

Full **creativity** flowers through student choice at each stage

1. *Problem or model formulation*
2. *The choice of methods to apply*
3. *The construction of solutions*
4. *Deciding which solutions are optimal*

6. Social Empowerment via Maths

- ***Mathematics is a political subject!***
- Maths should socially and politically empower students to be numerate critical citizens in society.
- **Critical mathematical citizenship** involves

  * *Ability to use maths in social and political activity, for the betterment of students themselves and for democratic society as a whole*
  * *Critically understanding the uses of mathematics in society*
  * *Identifying, interpreting and critiquing maths in social, commercial and political claims, from adverts and headlines to blogs and reports*
  * *Scrutinizing financial sector and government systems and procedures*
  * *Understanding the limits of validity of uses of maths, what decisions are concealed, and rejecting spurious or misleading claims.*

- **Every citizen needs these capabilities to defend democracy and the values of humanistic and civilised society**

Critical Mathematical Pedagogy

- To develop critical mathematical literacy and citizenship need to have:
  - *Extended **social maths projects***
  - *Students to **choose** topics that matter to them (environment, aid, social problems, etc)*
  - *Students to **choose** what maths tools & methods best suit chosen topics*
  - *Students to **discuss** methods and results in depth together and with teacher*

7. Appreciation of Mathematics

English has

- *Reading and writing (language skills & capabilities)*
- *Literature (Appreciating texts and their social and cultural place)*

Maths has

- *Calculating, Solving problems (maths skills & capabilities)*
- *Appreciating maths and its social and cultural place is **missing from school!***

Is maths only about doing?

I want to argue for the missing ***appreciation*** aspect of maths

What is Appreciation of Maths?

Maths more than calculating, solving and proving

School maths needs a broader appreciation of

- *Maths in culture, art and social life*
- *History of mathematics and maths in history*
- *Mathematics as a unique discipline*
- *Proof and how maths knowledge validated*
- *Controversies in philosophy of maths*
- *Big ideas of mathematics*

Maths in Culture, Art and Social Life

Students need to appreciate important role of maths in culture, art and society - including:

* ***Symmetry*** *in art and religious symbolism and in our appreciation of beauty*
* ***Algebraic equations*** *in scientific theories, e.g., Einstein's* $E=mc^2$ *underpins modern physics and cosmology*
* *Maths in* ***all aspects of daily life****, e.g., via commerce, economics, stock market, telecommunications, ICT*
* *Maths in representing, coding and displaying* ***all information***

However, need to recognise that maths is becoming invisible - hidden within the programmes and social systems that control complex technological society

History and Maths

- The fascinating history of maths sheds light on very abstract concepts – history makes school maths meaningful
- **But** there is a myth that *applied maths* arises from practical applications of *pure maths theory*
- **Actually** *maths first emerges as 'applied'* – only later do mathematicians abstract and purify concepts and methods and systematize theories
- Maths begins with ***ethnomathematics*** - informal culturally embedded maths concepts and skills found in every culture, past and present, rural and urban
- Of course once maths is invented some mathematicians study for **its own sake** - *purely out of interest* – e.g., Geometry in Greece, Algebra in Mid East, Number patterns in India, China and Europe
- But social needs in history have driven invention of most mathematical concepts, symbols, methods and problems.

Maths Invented for Social Needs

Country	Social Practice	Maths Topic Developed
Mesopotamia, India, Egypt	Tax, Accounting, Commerce	Numbers and Number Systems, Measures
Egypt	Surveying	Geometry
Medieval Europe & Mid East	Commerce, Navigation, Ballistics	Number Algorithms, Trigonometry, Mechanics
19th Century Europe	Agriculture, Medicine, Insurance	Statistics
Modern Europe	Mechanisation, Cryptography	Logic, Coding Theory

Maths as a Discipline

- Maths should be appreciated as a **distinctive** and **unique discipline**, with its own:
  - *Central branches and topics*
  - *Key concepts linking branches (see Big Ideas)*
  - *Powerful language – basic to other disciplines*
  - *Unique epistemology of maths – maths knowledge validated by deductive proof.*
  - *But proof has its limits – one case disproves, but finite number of cases cannot prove a generality*
- Need to know that much ***more to maths*** than **number** and just what is **taught in school**

Philosophy of Maths

- It is a shock to learn there are controversies in maths, such as over these questions:
  - ***Are the objects of maths discovered or invented?***
  - ***Is maths knowledge certain beyond all doubt?***
  - ***Did maths exist before people evolved?***
- **No right answers here** – although absolutist & superhuman views of maths can make some learners feel excluded
- Why not let students be **excited & stirred** by these ideas and discuss them?
- Discussing these controversies develops a more **critical attitude** to maths, and helps to counter automatic **attributions of certainty** to anything mathematical.

Big Ideas of Maths

Learners should be exposed to and gain *intuitive understanding* of big ideas in maths such as:

pattern, modelling, symmetry, structure, equivalence (invariance), proof, paradox, recursion, randomness, chaos, infinity.

Maths contains many of the deepest, most exciting ideas created by humankind.

These extend our imagination - *the scientific equivalent of poetry* - offering noble, aesthetic, and even spiritual experiences

Are these ideas suitable for school?

Are these aims concerning appreciation feasible for school? Are the concepts too complex?

No – not for firing the **imagination**. Even big ideas like infinity – 9 year olds happily discuss infinity of space and numbers

Maths too often seen as dull, routine, uninspiring

With c. 2000 hours of school maths time we can afford to:

- Develop *appreciation* to inspire and excite
- Grow *critical maths literacy* and *citizenship* for life
- Create expert *problem posers and solvers*
- Develop *confident learners* and users of maths

These VISIONARY AIMS can build more knowing confident students & citizens, and possibly a better society for all!

End of Slideshow