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# Summing Up : Australian Numeracy Performances, Practices, Programs And Possibilities.

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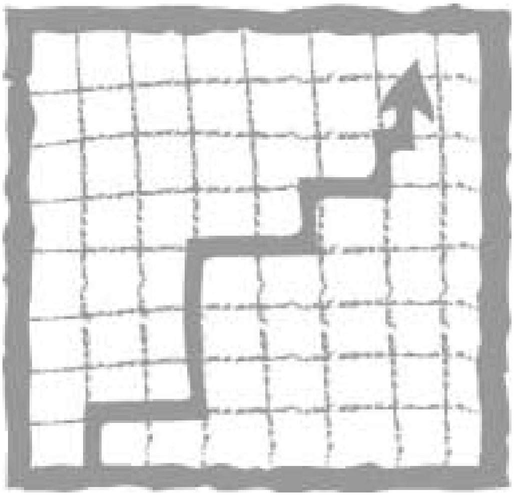

# Summing up: Australian numeracy performances, practices, programs and possibilities

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# INTRODUCTION

When the term ‘numeracy’ is used some questions that arise are ‘What is numeracy?’ or ‘Numeracy for whom?’ and even ‘Whose numeracy?’.

These questions highlight the difficulty the term has had in defining itself in a way that is clear and generally acceptable. While terms like ‘arithmetic’ and ‘mathematics’ are taken to be well-defined, the meaning of numeracy has varied greatly across time and place.

The following pages set out to clarify what has been meant by ‘numeracy’ in the past, as well as more recently, in order to better understand how numeracy teaching and learning can be improved in Australian classrooms. Defining numeracy inevitably assists understanding of what kinds of research may improve numeracy teaching and learning in our classrooms.

# WHAT IS NUMERACY?

Numeracy is a term that most people believe has a common, shared meaning. In fact, the term ‘numeracy’ (and its various off-spring such as numerate and innumeracy) can be used in different ways. These differences are of two kinds: the first relates to where and by whom the term is used, and the second to the changes in meaning of the term since its coining some 40 years ago.

Numeracy is originally a British term which still has little currency outside Britain and its former colonies, particularly Australia and New Zealand. Perversely, in the United States of America, the term ‘innumeracy’ is used, but not numeracy (Paulos, 1988). Educators in other parts of the world speak of either mathematics, or mathematical literacy, when wishing to convey what they think the British mean by numeracy.

The use of ‘*num*’ as part of *num*eracy, would seem to indicate also that numeracy is equivalent to *num*ber, but although some definitions do tend this way, many definitions of numeracy do not.

Part of the difficulty in capturing the meaning of the term numeracy stems from the fact that since its coining in the Crowther Report (1959) as a set of high-level skills and dispositions needed by managerial elites, definition of ‘numeracy’ has undergone dramatic changes. After the original definition by Crowther, numeracy made its next major appearance in the influential Cockcroft Report (Cockcroft, 1982) where it was re-defined as the skills and dispositions needed by ordinary people in work and daily life. This shift in meaning has made numeracy broader in its educational implications, a fact not lost on Australian educators, who, unlike their English colleagues, have not equated numeracy with number, a stance that has set mathematics educators in Australia on a different path.

In Australia, however, definitions of numeracy differ from education system to education system, a fact that makes communication and research between systems awkward. In an effort to lessen these difficulties and improve communication, the Commonwealth and states have agreed upon a shared definition of numeracy for Australian use. This definition stems from the Commonwealth-funded Numeracy Education Strategy Development Conference, which took place in 1997 under the joint auspices of the Education Department of Western Australia and the Australian Association of Mathematics Teachers. The published outcomes of the conference, *Numeracy = Everyone's Business*, focuses on the key elements central to a modern definition of numeracy: that

> numeracy involves … using … some mathematics … to achieve some purpose … in a particular context.
> (AAMT, 1997: 13)

The key underpinnings of this definition are that numeracy is essentially the effective use of mathematics

> to meet the general demands of life at home, in paid work, and for participation in community and civic life. Thus numeracy is:
>
> - distinct from literacy;
> - more than number sense;
> - not only school mathematics; and
> - cross-curricular.
>
> (AAMT, 1997: 39)

In its recommendations for future numeracy initiatives, the conference states that to be numerate 'is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life' (AAMT, 1997: 18), a definition that addresses the Cockcroft Report's concern that 'those who set out to make their pupils 'numerate' should pay attention to the wider aspects of numeracy and not to be content merely to develop the skills of computation' (Cockcroft, 1982: para 39).

This Australian definition, accepted by a wide range of mathematics and other educators in Australia, is much broader than, for example, the definition of numeracy advocated in England: that 'numeracy is the ability to process, communicate and interpret numerical information in a variety of contexts.' (Askew, Brown, Rhodes, Johnson & Wiliam, 1997: 4). Under the English definition there is a clear focus on skills with numbers. Other areas of mathematics and mathematical ideas do not enter the picture. More recently this definition was revised to include aspects of measurement and data handling (DfEE, 1998) but the impact of this change has yet to be seen.

Some educators however, argue that redefining numeracy in terms of 'more content' is neither sensible nor productive. Richard Noss, in his inaugural professorial lecture New Cultures, New Numeracies (Noss, 1997), has argued that while numeracy originally had been framed within a utilitarian culture, the nature of this culture has now changed necessitating a radical redefining of numeracy. Thus the defining of numeracy continues.

As reported earlier, not everyone uses the term 'numeracy'. Terms that are used to represent the principles and ideals akin to the Australian definition of numeracy range from the well-defined 'mathematical literacy' of the OECD Programme for International Student Assessment (PISA) project, to the less well-defined 'school mathematics' common in the United States. However, to exclude from this discussion, research and practice from those many countries where the term numeracy has no currency, would make our overview extremely narrow. In the light of this, it is necessary for the reader to bear in mind that 'numeracy' is being treated as a rather elastic term.

# PERFORMANCE OF AUSTRALIAN CHILDREN

The numeracy achievements of Australian children could be viewed from many different perspectives. The perspective taken here begins with a broad view — international — and narrows to focus upon specific groups, such as boys or non-English-speaking children. While international studies provide much information of an overall achievement nature, they also can identify the strengths and weaknesses of the children assessed and it is these aspects that are of interest here.

## International studies

### Third International Mathematics and Science Study

In the case of the Third International Mathematics and Science Study (TIMSS) some 8000 Australian children were involved nation-wide. The study was the most comprehensive international study of mathematics learning ever undertaken. Australia participated in TIMSS at three population levels: 9-year-olds, 13-year-olds and students in Year 12. Randomly-sampled schools and students from all states and territories and all education sectors took part. At each of the 9-year-old and 13-year-old levels, students were sampled from two adjacent year-levels containing the majority of the age group. These year-levels are referred to as the ‘upper’ and ‘lower’ years within each sample. In Australia, because of the different ages at which students start school state by state, Years 3 and 4 were sampled in some states and Years 4 and 5 in others to capture the 9-year-olds. Similarly, the 13-year-old sample involved Years 7 and 8 or Years 8 and 9.

Australia’s comparative results in mathematics in TIMSS were creditable, and the results summarised below are taken from the three international TIMSS reports (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith, 1996; Mullis, Martin, Beaton, Gonzales, Kelly & Smith, 1997; Mullis, Martin, Beaton, Gonzalez, Kelly & Smith, 1998).

From a numeracy standpoint, of particular interest in the Australian TIMSS results are:

- the relatively poor performance of the primary age students in ‘whole numbers’, suggesting a lack of computation skills that was borne out by an examination of performance on individual items (see Stacey, 1997);
- the better relative performance in ‘data representation and analysis’ at lower secondary level than at mid-primary level;
- the improvement in relative performance on ‘geometry’ between the 9-year-old sample and the 13-year-old sample;
- the upper grade 9-year-old sample scored relatively better in ‘geometry’ and relatively worse in ‘whole numbers’ and ‘fractions’ than in ‘measurement, estimation and number sense’, ‘patterns, relations and functions’ and ‘data representation and analysis’;
- the upper grade 13-year-old sample scored relatively better in ‘data representation and analysis’ and relatively worse in ‘geometry’ than they did in ‘fractions and number sense’, ‘algebra’ and ‘measurement’;

- the lower grade 13-year-old sample scored relatively better in ‘fractions and number sense’ and ‘measurement’ and relatively worse in ‘geometry’ than they did in the two other areas.

These results from TIMSS provide information at an international comparative level and also gives an insight into Australian numeracy strengths and weaknesses that can guide future research and action.

### Programme for International Student Assessment

The current OECD Programme for International Student Assessment (PISA) assessed 15-year-olds in August 2000 as part of a survey on students’ preparedness for adult life. While in the 2000 testing reading was the major focus and mathematics and science minor foci, in 2003 mathematics will be the major focus. One of the aims of the PISA study is to provide information at a policy level, to be able to answer the question of how well our schools prepare students for participating in society. In order to do this, PISA defines a ‘mathematical literacy’ which is:

> The capacity to identify, to understand, and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual’s current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen.
>
> (OECD, 2000: 10)

This focus on applying school mathematics to real-life situations is closer to numeracy than to school mathematics, and means that the results from PISA will give a picture of how students use their numeracy skills acquired during the years of compulsory schooling. A good indication of the numeracy-like focus of PISA can be gained from the content categories used in the PISA assessment materials for mathematical literacy. These categories include: mathematical thinking; mathematical argumentation; mathematical modelling; problem solving and posing; representation; communication; decoding and interpretation of formal language; solving equations; and knowing about and being able to use a variety of aids and tools to assist mathematical activity.

The results of the PISA research will give a picture of Australian students’ numeracy achievement as they leave compulsory schooling and begin life as constructive, concerned, and reflective citizens.

## Trends over time within Australia

Some international studies, in which Australia has participated, have been repeated and thus provide the opportunity for trends to be observed. Two of these are the group of IEA studies conducted over the past thirty years in mathematics at the 13-year-old level, and the ACER-conducted Longitudinal Surveys of Australian Youth which has assessed numeracy performance over a period of approximately twenty years for 14-year-olds.

### IEA studies

The International Association for the Evaluation of Educational Achievement (IEA) conducted the First IEA Mathematics Study (FIMS) in 1964. In this first study only government schools in New South Wales, Victoria, Queensland, Western Australia and Tasmania participated. The number of students involved was 4320. In the Second IEA

Mathematics Study (SIMS), conducted in 1978, non-government schools, the Australian Capital Territory and South Australia were also involved. The number of students involved was 5120. The Third International Mathematics and Science Study (TIMSS), conducted in 1994, included 7392 government and non-government students in all states and territories.

Rosier (1980) studied the changes in achievement occurring between the FIMS (1964) and SIMS (1978) for the 13-year-old and Year 12 cohorts. He reported, *inter alia*, that there had been a slight decline in the mathematical achievement of 13-year-olds (except in Western Australia) though not in the area of computational skills, and that there had been a visible growth in the achievements of Year 12 students over the intervening fourteen years. While the two IEA studies were at secondary level, Rosier observes that reasons for the observed decline in secondary achievement in some states, between 1964 and 1978, should:

> be sought earlier in the school life of the student. The results for Queensland tended to support this position, since the mean test scores for that State, for both 1964 and 1978 were very high, where this achievement is built on a very strong emphasis on mathematics in the primary school in Queensland.
> (p.198)

In a meta-analysis, by path-analysis, of the Australian data from the three IEA studies, Afrassa and Keeves (1999) reported that:

> The results of the path analyses for the three different data sets (FIMS, SIMS and TIMSS) have revealed that the home background of students, number of students in class and attitudes of students towards mathematics are student level factors influencing achievement in mathematics over the last 30 years. Time in learning mathematics did not show any influence both in 1964 and 1978. However, it influenced the 1994 students' achievement directly, the main reason perhaps being the inclusion of a new and strong manifest variable, frequency of mathematics homework in a week.
> (p.53)

### Longitudinal Surveys of Australian Youth

The Longitudinal Surveys of Australian Youth (LSAY) provide linked assessments of numeracy achievement over a period of approximately twenty years for samples of 14-year-olds. The tests used consisted of mostly multiple choice items, but they did make an attempt to cover applications of mathematics in every day situations. Items were classified as 'computational', 'practical' (strongly relating to everyday contexts, e.g. hours of opening of a chemist shop) or 'conceptual'. Random samples of students were drawn and were tested first in 1975, with analyses over time presented on a common scale.

Among 14-year-olds, the mean scores on the numeracy scale showed that there was no overall change in performance between 1975 and 1995. There was a small improvement over that time interval in the percentage of students attaining mastery, and a small decline was noted on the computational items. No consistent change was evident on the practical items and there was a slight improvement on conceptual items (Marks & Ainley, 1997).

### TIMSS-R

Comparisons in mathematics achievement over the four-year period from 1994 to 1998, for two cohorts of the same age (13-year-old students) and in terms of the same cohort progressing from age 9 to age 13, are possible through the repeat testing of lower secondary level students known as TIMSS-R (Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski & Smith, 2000). Australian students have maintained good standards between TIMSS (1994) and TIMSS-R (1998) testing. Although not statistically significant, Australia showed small increases in achievement in all areas. Australian average achievement in mathematics was *significantly higher* than the international average in fractions, number sense and proportionality; measurement; data representation, analysis and probability; and algebra but *not significantly different* from the international average in geometry. There was no difference in *average* mathematics achievement between Australian boys and girls.

### System-level trends

In 1988 the then Victorian Ministry of Education commissioned a study of the literacy and numeracy achievements of Victorian 10- and 14-year-old students (McGaw, Long, Morgan & Rosier, 1989). As part of this study links were made to similar studies (the Australian Studies in School Performance) conducted in 1975 and 1980. Results reported on performances in the three studies showed that between 1975 and 1988 10-year-olds maintained their level of performance. Noteworthy was the fact that there was:

> some evidence of improvement in the levels of achievement of the poorer performing students.
>
> (p.102)

On the other hand, the 14-year-olds had made a significant improvement in achievement between 1975 and 1980, but subsequently had declined in 1988 towards the 1975 level.

Trend data are also reported in several of the state-wide assessment programs. These use either mathematics or numeracy as a defining term, but the assessment tasks are similar. Some recent examples are the report on the 1998 monitoring of mathematics achievement at Years 3, 7 and 10 in the WA Monitoring Standards in Education (MSE) program (van Wyke, 1999), the report of the NSW Basic Skills Testing at Years 3 and 5 (Department of School Education, 1998) and the report of state-wide performance of students in Queensland on their programs in 1995, 1996 and 1997 (Queensland School Curriculum Council, 1998).

The Western Australian report showed that there was an improvement in the achievement of students at all year levels assessed between 1996 and 1998, and these differences were statistically significant (van Wyke, 1999). However, these improvements were not consistent for all students: for example, the performance of Year 3 Aboriginal and Torres Strait Islander students declined in this period.

The Queensland report showed increases in all three aspects of numeracy assessed (Number, Measurement and Space) from 1995 to 1996, and a generally smaller increase, or even a decrease, from 1996 to 1997. For every subgroup of students the 1997 mean score was higher than the 1995 mean score. Results were analysed for several groups of students and all followed the same pattern except for Indigenous students whose result in Measurement did not increase from 1995 to 1996.

The IEA reports would suggest that any observed decline, and hence the possibility of improvement, is dependent on two types of factors: those that can be addressed directly by system-wide actions (a strong primary level mathematics curriculum, class size and frequency of homework), and those that identify groups of students who may need special assistance (particular home background factors). Although some systems have reported improvement in numeracy achievements over time, in most cases the extent of the trend periods tend to be shorter than those of the IEA studies, and provide less compelling directions for further research at this stage.

While large-scale assessment results are in the main encouraging, some argue that there are confounding factors involved in interpreting the results. This argument has been made particularly in the case of mathematics items set within a context that requires a high level of verbal reasoning skills (i.e., reading). Rowe (1999), for example, commenting on the results of Tasmanian state-wide tests of literacy and numeracy at Years 3 and 7, says that:

> It is important to emphasize that the 1998 Year 7 [and Year 3] numeracy test items all had excessive requirements for high levels of verbal reasoning skills. As such, the composite constructs of Literacy and Numeracy are confounded – as evidenced by the strong positive correlation between the two variables … In such circumstances, it is vital that invalid inferences are not made about students' levels of achievement in mathematics (per se).
>
> (p.28)

Further he suggests that

> to minimize this problem in future monitoring projects, it is recommended that numeracy test items in each domain be included that place minimal demands on students' verbal reasoning 'abilities' and skills.
>
> (p.28)

Whether mathematics (and mathematics assessment) should be in context is an unresolved issue, but those involved in either constructing assessment tasks or interpreting the results of these, must take heed of Rowe's comments, lest unfounded strengths and weaknesses be ascribed to our students.

## Sex differences

It is common to find studies reporting results that show males performing at a significantly higher level in numeracy than females in many countries. However the TIMSS results showed that significantly different performance by sex, always in favour of males, occurred in only six of 24 countries for the 9-year-olds, in only seven of 41 countries for the 13-year-olds, but in 11 of 16 countries in advanced mathematics at final year secondary level. Australia was one of a very small number of countries where no sex difference in mathematics performance at any of these levels was found (Lokan, Ford & Greenwood, 1996; 1997; Lokan & Greenwood, 1998).

In published results from the state-wide assessments in Australia differences in performance between the sexes is usually slight and not statistically significant. For example, the results of state-wide assessment in New South Wales show that females performed slightly better than males in 'Number' at Years 3 and 6, at about the same level in

‘Measurement’ but slightly lower in ‘Space’ (Masters, Lokan, Doig, Khoo, Lindsey, Robinson & Zammit, 1990; Doig & Lokan, 1997). A detailed analysis of the New South Wales results across several years was undertaken by Barnes (Barnes, 1997) and confirmed the earlier findings.

A similar lack of difference has been found in Western Australia where, in 1992, females achieved at the same level as males in all areas of mathematics at Years 3 and 7, except for Space at Year 3 where the females performed better. Males significantly outperformed females in all areas except Space and Algebra at Year 10. In both of these areas there was no difference in performance between the Year 10 groups (Titmanis, Murphy, Cook, Brady & Brown, 1993). In 1996, Year 3 females again performed better than males in Space and, in Years 7 and 10, males performed significantly better than females in Measurement. In other strands at Year 10 there was no sex difference (van Wyke, 1998).

The 1998 Queensland report shows that the 1997 performances of males and females were similar in Number and Measurement, while in Space the performance of males was slightly above that of females. The report of the 1997 Multilevel Assessment Program in the Northern Territory (NT Board of Studies, 1998) showed that males’ performance at Year 4 and Year 6 was not significantly, though very slightly, higher than females’ in mathematics.

The overall picture from these Australian results is that performance differences between sexes in mathematics classrooms appear not to exist, as any differences found are small, and considered to be of no practical importance. However, despite these research findings, the stereotypical view that males are better at mathematics than females continues to persist, indicating a need for better dissemination of research findings in this area.

## Numeracy achievement of Indigenous and non-English-speaking students

The achievement of Indigenous Australians is of concern at all levels of schooling and in all states and territories. Analyses of state and territory assessments, and in TIMSS, shows that the acquisition of numeracy by Indigenous Australians is considerably behind that of their non-Indigenous counterparts. The report of the 1996 Western Australian Monitoring Standards in Education comments that ‘the performance of ATSI students continues to be a concern. In general terms, their performance at each year level was almost a full outcome level lower than the performance of the rest of the population’ (van Wyke, 1998: 6).

In NSW the 1997 state-wide assessment report, using longitudinal data, showed that the ATSI group demonstrated more growth in numeracy from Year 3 to Year 5, than any other group. The report of the 1995 to 1997 Queensland assessment program commented that the performance of Aboriginal and Torres Strait Islanders was ‘more than extremely below that of the rest of the population’ (Queensland School Curriculum Council, 1998: 18).

Dawe and Mulligan (1997) provided a thorough review of language factors in mathematics learning. Their review illustrates the major features of language in mathematics, drawing on the New South Wales Basic Skills Testing Program for examples. In their analyses, it was shown that correlations between language background, sex, word knowledge and socio-economic status and achievement were similar for each type of item. The gaps in achievement were more marked for the older Indigenous students, which has been noted in other studies (see for example, Masters, Lokan, Doig, Khoo, Lindsey, Robinson & Zammit, 1990). Of most concern were the percentages of Indigenous students who did not attempt to respond to items that required answers to be written rather than selected.

# PRACTICES IN NUMERACY

Effective practices in numeracy may be re-stated as 'what we do' (where the 'we' is classroom teachers) that is effective.

The studies below describe teachers' practices that range from those revealed by research into current practice, to practices based on research and specifically and directly encouraged in classrooms. It is interesting to note that despite differences in time and place, there are common themes running through all of these five studies.

## Effective teachers of numeracy

What makes a teacher effective in teaching numeracy is a key question for education employers as much as for providers of initial teacher-training and teacher professional development. The question of what makes for effective teaching in numeracy, however, has only recently been addressed directly. This was the purpose of a major research study conducted in England, the Effective Teachers of Numeracy Study (Askew, Brown, Rhodes, Johnson & Wiliam, 1997) and its findings are currently being re-examined and the research extended in a five-year longitudinal study.

In the Effective Teachers of Numeracy Study, teacher effectiveness in the English context was classified according to average gains of pupils in specially designed tests. The results of the study may be broken into two parts: one dealing with the classroom organisation of effective teachers, the other dealing with teachers' beliefs about teaching and mathematics. That there was no common form of classroom organisation used by effective teachers was a surprising finding, particularly given the organisational focus of the (English) National Numeracy Strategy later implemented. Effective and less effective teachers were found to be equally likely to use whole class, small group or individual approaches in organising their mathematics lessons.

On the other hand, teachers' beliefs about teaching and mathematics were a strong differentiating factor between highly effective and other teachers. Teachers in the study were interviewed about the educational orientations underlying their beliefs and attitudes to teaching, mathematics and styles of interaction with students. The results of these interviews led to the defining of three models of orientation that explained how teachers approached the teaching of numeracy. These orientations were defined as follows:

*connectionist* teachers – have beliefs and practices based on valuing students' methods, using students' understandings, and placing emphasis on making connections within mathematics.

*transmission* teachers – have beliefs and practices based on the central role of teaching, and a view of mathematics as a collection of discrete skills, conventions and procedures to be taught and practised.

*discovery* teachers – have beliefs based on the central role of learning, and a view of mathematics as being developed by students, particularly through interactions with concrete materials.

All but one of the highly effective teachers of numeracy were classified as connectionist, while teachers holding other orientations were all classified as being only moderately effective. The question that arises from this study is

‘How does one become a connectionist teacher?’. Background information collected during the study clearly links long-term professional development courses (ten days or more) that focus on children’s conceptions and strategies as the single most important correlate with connectionist teachers. However, whether connectionist teachers are ‘made’ by these courses, or whether connectionist teachers are more likely to participate in these courses is unclear.

The study also confirmed other UK and US research that found that neither mathematical qualifications nor initial training are strong factors in developing highly effective, or effective, teachers of numeracy.

As expected, these results from the Effective Teachers of Numeracy Study raised many questions and the independently initiated Leverhulme Numeracy Research Programme was expected, *inter alia*, to clarify the results of the study (Brown, 2000).

The Leverhulme Numeracy Research Programme is a five-year study that commenced in 1998. Results to date confirm some of the key results of the Effective Teachers of Numeracy Study. That is to say, there is no correlation between the proportion of whole class teaching, use of calculators or amount of homework and class gains in numeracy scores. Higher teacher qualifications in mathematics also appear to have no effect. On the other hand, the effect of longer term professional development on effective numeracy teaching has not been confirmed (Brown, 2000).

### Numeracy teaching practices

The study of teachers’ practices is believed to be a critical focus for research into effective numeracy teaching and learning, despite evidence that teacher and school effects typically account for less than 10 per cent of the variation in achievement (see for example, Creemers, 1997, cited in Brown, Askew, Baker, Denvir & Millett, 1998). Nevertheless calls for change in teaching practice continue, leading Stigler (American Federation of Teachers & National Centre for Educational Statistics, 1998) to declare ‘Let’s look at examples and let’s say exactly what it is about this [practice] that you’d like to see changed. That’s how we come to understand what good teaching is. We haven’t had this conversation in this country [the USA]’.

And this is exactly what Stigler did! In an innovative approach to examining classroom practice, undertaken as part of the Third International Mathematics and Science Study (TIMSS), the TIMSS Video Study (Stigler & Hiebert, 1997) observed Year 8 mathematics classrooms randomly selected from within the TIMSS sample in three countries: Germany (100 classrooms), Japan (50 classrooms) and the United States (81 classrooms). This form of data collection was previously unavailable on a large scale. The video-tape data were analysed according to frameworks that facilitated understanding of the teaching practice across languages, curriculum and cultures. Typical of the questions addressed by this analysis are: What kind of mathematics do students encounter?, Are mathematical processes and procedures developed?, What are students expected to do?, What is the teacher’s role? And How are lessons organised? (Stigler & Heibert, 1997).

The analysis of the video-data enabled Stigler, Gonzales, Kawanake, Knoll & Serrano (1999) to state that Japanese lessons, unlike those in the US, ‘include high-level mathematics, a clear focus on thinking and problem solving, and an emphasis on students deriving alternative solution methods and explaining their solutions' (vii).

A similar, follow-up international video study (TIMSS-R) is being conducted currently with eight countries involved. In each school one Year 8 mathematics and science lesson will be video-taped. The aims of the study are to:

- paint national-level portraits of mathematics and science teaching practice;
- develop new teaching research methods and tools for teacher professional development;
- stimulate and focus discussion of science and mathematics teaching practices among educators.

Eighty-three Australian secondary schools took part in the study which will also provide explanatory data on the contexts in which mathematics and science teaching and learning occurs.

### Mathematics Classrooms Functioning as Communities of Inquiry

At the primary school level, a smaller Australian Research Council-funded project Mathematics Classrooms Functioning as Communities of Inquiry: Models of Primary Practice was conducted in Victorian Government and independent schools in 1999 (Groves, Doig & Splitter, 2000). In this study, video and other data were collected from a stratified random sample of ten Year 3 and 4 classrooms in the state of Victoria. One mathematics lesson of approximately one hour's duration was video-taped in each of the ten classrooms and an outline of the aims for each lesson, as well as copies of any work-sheets used by the children, were collected. An analysis of the video-tapes was carried out, using a framework based on that developed by Schmidt, Jorde, Cogan, Barrier, Moser, Shimizu, Sawada, Valverde, McKnight, Prawat, Wiley, Raizen, Britton & Wolfe (1996), who use the term 'characteristic pedagogical flow' to describe recurrent patterns of observable characteristics in a set of lessons. Based on observations, field-notes and this analysis, three vignettes were produced, representing the contrasting characteristic pedagogical flows captured on the Victorian video-tapes.

These vignettes were used as a stimulus for three separate four-hour focus group meetings for randomly selected teachers, principals and mathematics teacher educators and consultants. Discussions were based around the findings from the analysis of the ten Australian video-tapes and a viewing of the vignettes. The last fifteen minutes of each meeting were devoted to participants completing written responses to a list of 'prompts' in order to provide data on individual views.

The major feature identified by participants was that in current primary mathematics teaching practice there was a lack of conceptual focus in the lessons. Although the number of people in the study was small, their combined experience, and the fact that they were selected at random, gives a good indication of the broader situation.

### Early Numeracy Research Project

The Victorian Early Numeracy Research Project is a collaboration between the Department of Employment, Education and Training (DEET), the Australian Catholic University, Monash University, the Catholic Education Office (Melbourne), and the Association of Independent Schools of Victoria. The study is expected to be completed in the seventy project schools by 2002 (Clarke, 1999).

From a review of the literature, the project team has developed a framework of key growth points in children's numeracy learning to allow planning for teaching as well as providing a basis for identifying and describing growth in numeracy. Initially (1999) the project focused on counting, place value, addition and subtraction, multiplication and division, time, length and mass. Spatial aspects were added to the framework in 2000.

In terms of change to teachers' practice, the Early Numeracy Research Project teachers were asked to report changes in their practice, and an analysis of these reports revealed several common themes. These included:

- more focused teaching (in relation to growth points);
- greater use of open-ended questions;
- giving children more time to explore concepts;
- providing more chance for children to share strategies used in solving problems;
- offering greater challenge to children, as a consequence of higher expectations;
- greater emphasis on 'pulling it together' at the end of a lesson;
- more emphasis on links and connections between mathematical ideas and between classroom mathematics and 'real life mathematics';
- less emphasis on formal recording and algorithms; allowing a variety of recording styles.

(Clarke, 2000: 5-6)

The key elements emerging from these studies examining effective numeracy teaching practices are a clear focus on concepts and thinking, an emphasis on valuing children's strategies, and encouraging children to share their strategies and solutions.

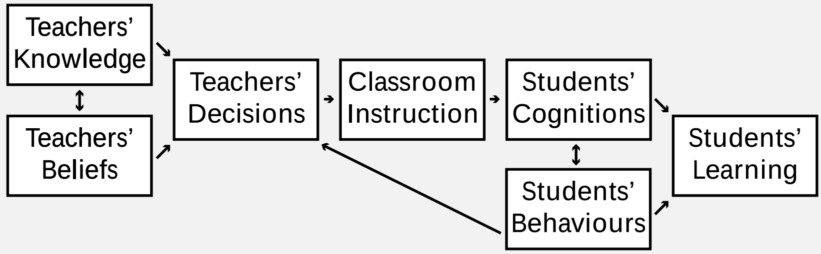
### Belief systems

Studies of teachers' conceptions are important in the context of effective practices in numeracy and, as Thompson (1992) concludes in her synthesis of the research on teachers' beliefs:

> no description of mathematics teaching and learning is adequate and complete unless it includes consideration of the beliefs and intentions of teachers and students.
> (p.142)

Teachers' beliefs about what mathematics is and how mathematics should be taught have been investigated as key factors in children's learning for more than a decade, and the work of researchers in the United States has been instrumental in raising educators' awareness of the issue.

The Cognitively Guided Instruction (CGI) model of mathematics teaching is founded on the principle that teachers' pedagogical decisions should be made on the basis of a cognitive science understanding of how children learn particular content (Carpenter & Fennema, 1988; Fennema, Carpenter & Peterson, 1989). In the CGI model, the teacher's decisions are regarded as being affected by their knowledge of mathematics and children's mathematical development, and the teacher's beliefs about each of these. The diagram below shows the CGI model with these two key factors affecting teacher decision-making bolded at the extreme left.




*(Fennema, Carpenter & Peterson, 1989: 180).*

Since the early research of Fennema, Carpenter and Peterson the CGI model has been adopted and adapted in many later studies. Numeracy researchers in Australia and the United Kingdom have included teacher beliefs as a core element of their research over the last decade, particularly those beliefs about what constitutes effective numeracy teaching.

For example, what Australian primary teachers believe to be effective teaching was examined through looking at student outcomes in the National Board of Employment, Education and Training's commissioned report *The Elements of Successful Student Outcomes: Views from Upper Primary Classroom Teachers,* (NBEET, 1995). In this study, upper primary teachers were surveyed on a range of factors thought to affect children's achievements. Of thirteen elements suggested by the survey makers as having an impact on achievement, respondents selected six as being the most critical factors and of these six, pedagogy was believed to be critical by 11 per cent of respondents and curriculum by 10 per cent.

The pedagogical practices believed by survey respondents to be of most benefit were the use of a variety of approaches, clear communication of expectations, diagnosis of individual needs, and collaborative learning approaches. Curriculum-related beliefs of the respondents were that literacy was by far the most important influence on children's achievements (19 per cent support), but a numeracy focus was also believed important (12 per cent support). This study, although not intending to focus on numeracy, does shed some light on Australian teachers' views of the importance of numeracy and associated effective teaching practices.

Following the results of the Effective Teachers of Numeracy Study (Askew, Brown, Rhodes, Johnson & Wiliam, 1997) a focused study, the Leverhulme Numeracy Research Programme (Brown, 2000), is attempting a detailed analysis of the characteristics of effective teaching practice. As Brown points out '[effective teaching] seems to be strongly related to the orientation of teachers, including beliefs and pedagogical content knowledge underpinning their practice, although not their formal subject qualification' (Brown, 2000: 6).

Askew, too, summarises the findings of the Effective Teachers of Numeracy Study as showing 'an association between teachers' beliefs about how best to teach numeracy and pupil gains' (Askew, 2000: 2).

# PROGRAMS FOR IMPROVING NUMERACY LEARNING

Programs designed to improve numeracy learning (and teaching) usually have an initial identification phase followed by teaching derived from research findings. These programs use data from the identification phase to align specific teaching and learning strategies to the needs of individual children. In some cases, however, a more general approach is used and all children involved, with the consequence that benefits of the program are differential across the ability range of the children. These large-scale numeracy strategies are widely debated, and arguments both for and against them continue. Despite this several large-scale projects exist, some of which are described in the next section.

## System-wide approaches to improving numeracy learning

### The National Council of Teachers of Mathematics 'Standards'

In an attempt to prepare the education systems of the United States for the twenty-first century, the National Council of Teachers of Mathematics published *An Agenda for Action* (NCTM, 1980) which initiated a call for reform. Throughout the 1980s the NCTM published articles, books and classroom support materials to assist teachers reform their classrooms. A significant set of publications that formed part of this reform movement was the 'standards'. These included the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), the *Professional Standards for Teaching Mathematics* (NCTM, 1991), and the *Assessment Standards for School Mathematics* (NCTM, 1995). Despite a massive effort spanning more than two decades, anti-reform groups, such as Mathematically Correct, have mounted a 'maths war' against the reforms and this has had a considerable effect, particularly with the release of the Third International Mathematics and Science Study (TIMSS) results. In an effort to reduce criticism of the 'standards' and provide support for reform teachers, the NCTM issued *Commonsense Facts to Clear the Air* (NCTM, 2000). These 'facts' included a defence of the aims of the reform, which went far beyond an emphasis on simple arithmetic, the 'basics' of the anti-reform movement.

> FACT #1: School mathematics must meet the needs of a much greater proportion of students than it has in the past. NCTM advocates a mathematics curriculum that meets the needs of ALL students, without short-changing any student.
>
> Why? Because for much of history mathematics has been an effective "sorter" of human talent: few "got it", some mastered little more than arithmetic basics, and many were left far behind. Today, however, changes in the workplace, the demands of effective citizenship, and the mathematizing of so much of our lives requires that school mathematics empower all students. Meeting this goal of building mathematics programs that empower all students implies changes in curricular expectations for students as well as in instructional practices. Quality mathematics for all is an enriched mathematics, not a watered-down mathematics.

Learning outcomes from the 'standards' movement have been hotly debated in the US and no clear picture has emerged as to the effectiveness, or otherwise, of the proposed reforms. As with other optional alternatives, this may be due in part to the lack of a clear understanding by teachers of what the day-to-day practices of a 'reform teacher' are.

### National Numeracy Strategy

Probably the best known of system-initiated classroom-based numeracy projects aimed at improving the numeracy of all children is the National Numeracy Strategy (NNS) developed for England and Wales. Still in its infancy, the NNS has yet to prove itself, and the outcomes of the national testing over the next few years will be awaited with keen interest.

The National Numeracy Strategy is the most influential large-scale initiative in school mathematics for many years. The term numeracy is mainly now attached to the NNS initiative to raise standards in mathematics teaching through a collection of methods and especially 'the numeracy hour' in Primary schools, in which lessons are constructed to a template, with objectives drawn from the NNS framework, and using 'direct teaching'. The NNS is currently for Secondary schools, but it is to extend to include all schools by the end of 2001.

In the strategy, numeracy is defined as:

> knowing about numbers and number operations. More than this, it requires an ability and inclination to solve numerical problems, including those involving money or measures. It also demands familiarity with the ways in which numerical information is gathered by counting and measuring, and is presented in graphs, charts and tables.
>
> (DfEE, 1998: para 10, p 6)

In the classroom children are to achieve numeracy through the agency of the 'numeracy hour' although an hour is not mandatory. The single requirement is for a daily, dedicated mathematics lesson. The NNS does, however, suggest a framework for these daily lessons. In *The Implementation of the National Numeracy Strategy: The Final Report of the Numeracy Task Force* (DfEE, 1998) Chapter 1, paragraph 27 the Framework is put this way:

> Primary schools, and, where appropriate, special schools, should use the practices that we have identified as effective ways of raising standards of mathematics, by:
>
> - reviewing, and adjusting where necessary, their planning and teaching practices in mathematics, using the Framework for teaching as a tool to help them do so;
> - teaching all their pupils a daily 45 to 60 minute mathematics lesson;
> - teaching mathematics to all pupils within a class at the same time, with a high proportion of lessons concentrating on the development of numeracy skills;
> - teaching mathematics to the whole class or to groups for a high proportion of the lesson, promoting participation from, and co-operation between, pupils;
> - including oral and mental work within each daily mathematics lesson; and
> - providing regular mathematical activities and exercises that pupils can do at home.

Further detail is in paragraph 32 where the organisation of every daily numeracy lesson is set out. The following shows the template for such a lesson.

*Clear Start to Lesson*

*Whole class*

- *mental and oral work to rehearse and sharpen skills*

*About 5 to 10 minutes*

*Main Teaching and Pupil Activities*

*Whole class/Groups/Pairs/Individuals*

- *clear objectives shared with pupils interactive/direct teaching input*
- *pupils clear about what to do next*
- *practical and/or written work on the same theme for all the class*
- *group work differentiated by levels of difficulty, with focused teaching of 1 or 2 groups for part of the time*
- *continued interaction and intervention*
- *misconceptions identified*

*About 30 to 40 minutes*

*Plenary*

*Whole class*

1 *feedback from children to identify progress and sort misconceptions*

2 *summary of key ideas, what to remember links made to other work*

3 *next steps discussed*

4 *work set to do at home*

*About 5 to 10 minutes*

To facilitate the NNS every Primary school has a numeracy co-ordinator, who is supported by numeracy consultants based in the Local Education Authorities. Other initiatives include ‘key maths teachers’, who conduct model lessons for other teachers to visit and observe. Training packages available include video exemplars of the NNS lesson plan, and special guidance for teaching able children. Summer schools are being offered in many regions to children to ensure that they begin Secondary school with an adequate mathematics proficiency at National Curriculum Level 4.

The authors of the National Numeracy Strategy claim that it is based on research evidence of methods that are effective in raising achievement in numeracy. This is a bold claim, and one that needs substantiating. In their review of effective numeracy strategies underpinning the NNS, Brown, Askew, Baker, Denvir and Millet (1998) examine the evidence for, and against, strategies employed in the NNS. Much of the evidence derives from the Third International Mathematics and Science Study (TIMSS) and work carried out by researchers in the ‘school effectiveness’ area.

As the authors point out ‘research findings are sometimes equivocal and allow differences of interpretations’ (p.378) and when taken with the ‘many practical constraints on policy which are likely to over-ride empirical evidence’ (p.378) it is virtually impossible to say with any certainty what strategies may work. Since the NNS itself is in a sense a large-scale research project, it is to be hoped that investigations into its methods and results may provide less ambiguous findings than are currently available.

Felgate, Minnis and Schagen (2000) conducted an extensive study of the impact of the NNS by assessing the numeracy achievement of nearly ninety thousand pupils in the United Kingdom. Students in Years 3, 4 and 5 were tested on three occasions and their performance and progress measured. Using multi-level analysis of the test results, linked to several background variables, the study has shed some light on factors related to achievement and progress in numeracy. Initial findings show some evidence, although equivocal, of students in larger classes progressing less than expected, and that girls made less progress than boys.

### The Victorian Numeracy Program

Rather than a single strategy, the Early Years Numeracy Program is a network of related strategies that form a system-wide approach to enhancing numeracy practices at the Prep-4 level. The Victorian Department of Education, Employment and Training (DEET) Early Years Numeracy Program has four major aspects (Early Years of Schooling Branch, 1999).

The Structured Classroom Program is modelled on the UK National Numeracy Strategy, particularly the emphasis on a daily ‘numeracy hour’ and the structured nature of this ‘hour’. Within the lesson is an initial ‘tools’ session (counting and mental mathematics practice) followed by whole class and small group teaching (of the core mathematics for the lesson). The lesson concludes with a whole class reflection on the mathematics learnt and strategies used. Although the Structured Classroom Program is not mandatory, schools are strongly encouraged and supported in adopting this approach through local numeracy advisers.

The second aspect of the Early Years Numeracy Program is the Additional Assistance program, in which early identification of children ‘at risk’ and strong parent-school links are fundamental. Further, the third aspect of the Numeracy Program is Parent Participation, based on developing a culture that encourages parents, and teachers, to see that education is a partnership between home and school. Professional Development for Teachers takes a key role in the Numeracy Program as it provides teachers with opportunities to develop the skills and understandings needed to successfully implement the Early Years Numeracy Program.

Other parts of the Early Years Numeracy Program is the SOFNet Program ‘Maths Beyond the School Gate’ that combines television broadcasts and interactive satellite communication. The lessons in this program model the Structured Classroom Program, while membership of an Early Numeracy Network provides continuing professional support for classroom teachers by classroom teachers.

Unlike other numeracy strategies, the Early Numeracy Network is not a single solution but, as its name suggests, is a network of complementary strategies, one supporting another. This rather complex approach has been successful in early years literacy.

### Count Me In Too

The Count Me In Too program is based on the research and practices of Wright (1991a, 1994) and Steffe (Steffe, Cobb & Von Glasersfeld, 1988). Implementation of the program in all New South Wales government schools is expected to be completed by 2003, and has been adopted by schools in New Zealand as well. The program is an extension of the Count Me In professional development materials and is adapted from the

Mathematics Recovery Programme (Wright, 1999). Teachers undertake professional development to learn about the counting stages defined by Steffe and to examine activities based on these stages. Teachers view video-clips in which the counting stages are high-lighted and teaching strategies implemented. Teachers explore their own students' placement within the sequence of counting stages and trial recommended classroom strategies.

## Research approaches to improving numeracy learning

Not all studies of numeracy are system-wide, and in the United States there have been several studies that have attempted to implement alternative approaches to numeracy teaching and learning. While the results of these studies are valuable to Australian educators, the American context of the studies must be borne in mind when considering the implications for teachers and students here. Each of the three studies described below has a different basis for the way in which they approached the problem of improving teaching and learning in school mathematics.

### The Connected Mathematics Program

The Connected Mathematics Program (CMP) from Michigan State University began in 1991 and continued to 1997 and was a result of the NCTM 'standards'. The CMP aimed to develop a complete mathematics curriculum for Years 6 to 8 (the middle years of school) based on the 'standards', which in the CMP context meant that students were to learn mathematics through investigations, wherein groups of students actively explored mathematical concepts and applications by solving problems, making conjectures, discussing solutions and generalising their findings. Teachers, likewise, were to change the way they taught to be more in-line with the NCTM suggested reforms for teaching. In brief, lessons were structured around three elements: launch (a focus on the mathematics in the lesson), explore (students investigate the problem), and summarise (a discussion drawing together strategies, solutions and mathematical connections). Throughout the lesson students are pushed to extract the important mathematics and to connect and generalise what they have learnt (Lambdin & Lappan, 1997).

Results from this six-year project that involved 2500 students (half in CMP classrooms and half in control classrooms) (Hoover, Zawojewski & Ridgway, 1997) showed that on the Balanced Assessment Project small group performance tasks the CMP students performed better than the students not in the program on:

> challenging open-response items that emphasize reasoning, communication, connections and problem solving.
>
> (p.12)

However the CMP students also made gains on a traditional multiple-choice test instrument, the Iowa Test of Basic Skills (ITBS), and these gains were comparable to other comparison groups. The authors list some caveats when reporting their findings, particularly with respect to rater effects in scoring open-response items, and the samples used for grade norm development in the ITBS.

However there was a clear difference in how effectively the two tests differentiated performance in mathematical thinking, a fact that has implications for assessment in numeracy at least in the middle years of schooling.

## MathWings

Robert Slavin has been proponent of co-operative learning for two decades, and has applied his research findings from studies of co-operative learning in literacy to mathematics. The Roots and Wings program includes MathWings which is based on the Success for All literacy program. The basis of a MathWings lesson is two-fold: an *action mathematics* unit which is a whole class focus on a major concept, and *power mathematics* units that are individualised and have a focus on computation and applications. These two unit types are embedded in a co-operative learning structure with curriculum content and goals based on the National Council of Teachers of Mathematics 'standards' (1989, 1991, 1995). However, to become a Roots and Wings school, it must have a long-term commitment to co-operative learning by at least 80 per cent of the staff. Slavin (1999) claims that the Success for All and the Roots and Wings have been successfully implemented in hundreds of US schools as well as internationally, and does not depend on:

> charismatic leadership, extraordinary funding, or other hard-to-replicate features
>
> (p.78)

## Cognitively Guided Instruction

When Shulman first defined teachers' pedagogical content knowledge in the mid-1980s it was as consisting of teachers' knowledge of ways of making subject-matter comprehensible to students, and of understanding of students' thinking, particularly misconceptions.

Pedagogical content knowledge, particularly an understanding of students' thinking, was taken as the basis for a teacher development program, Cognitively Guided Instruction (CGI), by Carpenter and Fennema in the early 1990s. The CGI program is based on research from cognitive science and on the National Council of Teachers of Mathematics 'standards' (1989, 1991, 1995), in particular the assumption that children construct their own knowledge (Carpenter, Fennema & Franke, 1996). Interestingly, the CGI approach mirrors a core element of the Japanese pedagogical approach. That is, teachers have a good knowledge of students' thinking and solution strategies when working on mathematical problems.

CGI is not a prescriptive program for teaching but instead a teacher uses CGI principles when teaching. According to Fennema and her co-workers, teachers who act in this way:

> (a) believe that their understanding of children's thinking is a critical component of instructional planning, (b) facilitate children's problem solving and discussions of children's thinking, (c) listen to their children and question them until the children's thinking becomes clearer, and (d) are willing and able to make instructional decisions that are appropriate to the mathematical needs of their children
>
> Fennema, Carpenter, Franke, Levi, Jacobs & Empson (1996)

These principles are applied differentially as the level of teachers' mathematical knowledge and pedagogical expertise vary from individual to individual. According to Vacc and Bright:

> Considerable personal reflection on one's beliefs and behaviour would seem to be necessary for one to develop coherent pedagogy.
>
> (Vacc & Bright, 1999: 106)

While considerable research, and teacher professional development effort has been necessary for CGI to exist, this has been worthwhile as the correlation between children's achievement and a teacher's use of a CGI approach is significant for problem solving, solution of addition and subtraction word problems and recall of number facts (Vacc & Bright, 1999).

## Programs designed for specific groups

In 1997 all state, territory and Commonwealth Education Ministers agreed on a national goal which stated:

> that every child leaving primary school should be numerate, and be able to read, write and spell at an appropriate level.
>
> (DETYA, 2000)

and

> To help support the achievement of these goals, Ministers also agreed on a set of key priorities for school education. These form the National Literacy and Numeracy Plan and provide a clear framework for the improvement of school literacy and numeracy standards in Australia.
>
> The main focus areas of the National Plan are: early assessment and intervention for students at risk of not achieving minimum required standards, the development of agreed national benchmarks in Years 3, 5 and 7, the assessment of students progress against benchmarks, national reporting of benchmarking data and the provision of professional development for teachers to support implementation of the National Plan.
>
> (DETYA, 2000)

As with the National Council of Teachers of Mathematics' 'standards' the Australian benchmarks are designed to provide a set of goals to aim for in numeracy. While some argue that the benchmarks set a minimum standard, a low target, to which 'standards' will fall, others look at the notion of 'every child leaving primary school should be numerate' and focus on the 'every child'. The programs and research reported below give an indication of the efforts being made to ensure that 'every child' achieves numeracy.

### Australian numeracy programs

Many education systems maintain special programs for groups of children who are deemed to be in need of some form of special assistance or treatment. In some cases funding for these programs is provided by the Commonwealth, in other cases they are self-funded by the system.

In an attempt to map current programs of this nature, the Australian Council for Educational Research conducted a telephone and facsimile survey of the provisions of the government, and Catholic education systems within Australia. Questions in this survey addressed the following aspects of mathematics provision and evaluation: system monitoring, special initiatives, special assistance programs, students at risk, equity issues, Indigenous students, and gifted and talented students, and the eighty pages of responses to the survey (Doig & Underwood, 2000) show the wide range of strategies currently being implemented.

The next sections provide details of programs in numeracy on two of these aspects of mathematics provision: Indigenous students, and those with learning difficulties.

Programs to increase educational opportunities for Indigenous students exist in all states and territories, but the achievements of Indigenous students who participate in such programs is often hidden when state-wide testing programs are the means of assessing numeracy achievement. However the report of the survey of non-capital Strategic Results Projects (SRP) of the Indigenous Education Strategic Initiatives Programme has revealed a wealth of achievement by Indigenous students. In *What works? Explorations in Improving Outcomes for Indigenous Students* (McRae, Ainsworth, Cumming, Hughes, Mackay, Price, Rowland, Warhurst, Woods & Zbar, 2000) the extent of Strategic Results Projects aimed at improving the educational opportunities for Indigenous students is laid out in great detail. In this survey of non-capital SRPs McRae et al. found:

> a series of experiments on a very large scale … a gold mine for policy-makers and practitioners alike.
>
> (p.1)

These 320 projects, or experiments, were in pre-schools, schools and in the VET sector, and related to literacy, numeracy, vocational education, as well as other areas of education. In essence these projects show what can be achieved in 'a relatively short space of time through concerted efforts'. (DETYA, 1997 cited in McRae et al. 2000: p.12)

The position of many Indigenous students in relation to numeracy learning is exemplified by the following:

> At the beginning of the program a particular student, typical of the students in the target group, could count forwards by ones to one hundred and by twos to twenty. She could not count backwards at all … she could not answer such questions as: When you are counting by twos, what number do you say before twelve? What number is two more than eight?
>
> (McRae et al. 2000: p.98)

However, typical of the results reported from these SRPs are the results at one site where:

> The target was to achieve 80 per cent of students moving one or more levels on the national Mathematics Profile. Eight of 11 students (72 per cent) did so.
>
> (McRae et al. 2000: p.93)

Again, at a site that focused on numeracy at the primary school level, 15 of 18 students progressed through one or more levels of the national Mathematics Profile in nine months, where each level represents approximately 18 months progress (p.153)

On a different scale is Improving Numeracy for Indigenous Students in Secondary Schools (INISSS). The major objective of this Tasmanian program is to improve numeracy outcomes for all students, but particularly Aboriginal students, in the middle years of schooling through a program of intensive teacher professional development (Callingham, 1999). The professional development program is based around the use of innovative tasks that pose realistic, intriguing and mathematically rich problems for students to solve. The results of this project to date show that 'the program appears to have met its goal of improving numeracy outcomes for all students, but particularly those of Aboriginal students' (Callingham, 1999: 3).

While both the McRae et al. report and the Tasmanian project give some indication of the type of programs aimed at improving educational outcomes for Indigenous students, both indicate that such targetted programs can have positive results. Indeed, Malin (1998) suggests that:

> the teacher who will be most successful will be those who have high expectations of Aboriginal and Torres Strait Islander students, who understands their students well, and who see themselves as learners, also open to new understandings from both their students and the parents of their students. (p.351)

### Students with learning difficulties

In March 1997 state, territory and Commonwealth Education Ministers agreed on national goals for education.

To help support the achievement of these goals, Ministers also agreed on a set of key priorities for school education, among which was:

> early assessment and intervention for students at risk of not achieving minimum required standards.
> (DETYA, 2000)

However, in their comprehensive report on literacy and numeracy learning for students with learning difficulties, *Mapping the Territory — Primary Students with Learning Difficulties: Literacy and numeracy*, Louden, W., Chan, L. K S., Elkins, J., Greaves, D., House, H., Milton, M., Nichols, S., Rivalland, J., Rohl, M. & Van Kraayenoord, C. (2000: Vol 1, 35) claim that over half the schools in their survey of 377 schools estimated that more than 10 per cent of their students were experiencing difficulties in numeracy.

While 71 per cent of principals said that numeracy was assessed in their schools, only 14 per cent of principals claimed that their schools had programs to support children with difficulties in numeracy. The report also suggests that there is:

> far less emphasis in schools on addressing the numeracy needs of children than on addressing literacy needs, even though numeracy needs are being identified.
> (Louden, et al. Vol 2, p.37)

An issue related to programs is that of training. According to Louden et al.:

> only around half of the schools in the survey indicated that they had on staff a specialist teacher with training in the area of learning difficulties, and in some schools there were no teachers at all with any training in the area.
> (Vol 2, p.38)

Of those schools that did indicate they were operating a numeracy support program, the most frequently mentioned were Support-a-Maths Learner, Numeracy for All and Count Me In Too.

Private providers of programs for students with learning difficulties in numeracy were also surveyed by Louden and his colleagues. These providers regarded as critical a formal diagnostic process followed by a report to parents and teachers that provided a rationale for teaching and behaviour management strategies. Among the many specific teaching strategies suggested were:

- frequent one-to-one teaching for new learning in a quiet setting;
- breaking down the activity or lesson into manageable parts;
- use of multi-modality techniques for learning (see, hear, do);
- provision of reinforcement in a variety of situations;
- implementing structured planning of activity;

- positive feedback at time of action;
- practical experiences relevant to the student;
- access to concrete materials for mathematics;
- provision of extra time;
- provide the student with successful experiences.

(Louden et al. Vol 1, p.12-13)

It is to be hoped that the release of reports such as *Mapping the Territory — Primary Students with Learning Difficulties: Literacy and Numeracy*, will increase awareness of the needs of children with difficulties in numeracy and lead to an increase in support in the next few years as the impact of the Commonwealth's Literacy and Numeracy Plan is felt. A significant issue may well be the development and wider use of intervention programs such as those currently being implemented.

## Identification of children 'at risk'

In contrast to broad strategies for improving numeracy learning, there are strategies that target individuals 'at risk'. This focus on identifying children whose achievement is considered below expectation may be the dominant, or only, facet to some strategies.

Identification of problems that children are having in numeracy fall within the area of diagnostic assessment, and the strategies outlined here include those using data from state-wide assessments, those forming part of numeracy intervention programs, and those used by teachers and psychologists working with individuals who are considered to be 'at risk'.

Diagnostic assessments are most often carried out on an individual basis, though states and territories with cohort testing programs often use results on those as a further opportunity to flag potential problems. The ACT uses the results from its cohort testing program to identify the lowest achieving 20 per cent of students in Years 3, 5, 7 and 9. New South Wales has a broad screening process, used by classroom teachers, based on the locally developed Schedule of Early Number Assessment (part of the Count Me In Too package) that is used for this purpose.

Victoria has an Early Years Numeracy Program, a component of which is the identification, through comparison with developmental stages, of 'at risk' students. Victorian schools use, *inter alia*, the New Zealand developed School Entry Assessment (SEA); a series of nationally (New Zealand) standardised performance tasks (Ministry of Education, 1997). The numeracy task, Check Out, is in the form of a shopping game. All children entering school are assessed with the SEA kit in their first two months at school, and within the context of the regular classroom. Check Out is administered individually by classroom teachers, who then interpret the results in terms of their curriculum frameworks.

In South Australia teachers assess children's numeracy through observation during normal classroom teaching and also through using specially developed assessment tools. The program is for all students but it will enable 'at risk' students to be identified and then helped. Western Australia has implemented a Students at Educational Risk program, in which teachers develop profiles of students' achievements and use these in relation to typical expectations to identify students who need additional support. The First Steps program has recently been expanded (from literacy) to include numeracy.

Tasmania began its Flying Start program in 1997. While literacy and other areas are also part of Flying Start, there is an emphasis on numeracy skills based on Wright's Count Me In Too materials. The NT has developed its own Assessment in the Early Years, a guide for teachers on strategies for identifying students at risk of not achieving at appropriate benchmark levels.

## Intervention programs

Intervention programs are usually of two parts: the diagnostic phase where children 'at risk' are identified, and a teaching phase where appropriate action is taken at an individual level. A side effect of this second phase is often change in teacher practice. The following sections give details of some of the most prominent intervention programs currently in use, together with the research basis of each program.

### Year 2 Diagnostic Net

The Year 2 Diagnostic Net used in Queensland is based upon two assessment approaches. First, detailed descriptive continua of mathematical development are provided. These continua are focused on number, space and measurement, and are divided into key phases that identify significant milestones in development. Hence, they are said to 'map' a child's mathematical development. Teachers in the early years are required to observe their students, and record their observations using a checklist of key indicators.

The second aspect of the Year 2 Diagnostic Net is 'validation'. In addition to the observation of students' mathematical development, teachers are also required to use a set of 'validation' tasks provided by the State Department of Education. These assessment tasks are designed to provide a 'validation' of the teacher's judgements that had been based solely on observation.

Children who are deemed to be 'at risk' are then provided with a suitable intervention program. As the Year 2 Diagnostic Net developmental continua are inter-linked with the Queensland Year 1 to 10 mathematics syllabus and resource documents, these provide a basis for any program of intervention that teachers may plan to implement based on a child's performance in the Year 2 Diagnostic Net developmental mapping.

Schools report to the parents of each Year 1 and Year 2 child in a standard report format. This report describes the phases of development (in literacy and numeracy) and indicates in what phase the child is operating. The reports are followed by parent-teacher interviews where an individual child's development can be discussed in more detail.

### Mathematics Recovery program

Mathematics Recovery started in 1992 as a three-year collaborative research project in north-eastern New South Wales, jointly funded through the Australian Research Council, New South Wales regional government and the Catholic school system (Wright, Stanger, Cowper, & Dyson, 1996). The program, for selected first-grade children, is a long-term, individualised teaching program with the aim of advancing the students' arithmetical learning to the point where they may return to the regular classroom.

The program is based on the research and practices of Wright (1991a, 1994) and Steffe (Steffe, Cobb & Von Glasersfeld, 1988) and is based on a diagnostic interview protocol used by teachers with individual students.

The results of each student's interview are related to a learning framework based on research on children's number development. After the initial interview, continuing assessment forms part of the teaching-learning process.

The six counting stages also form the basis of the teaching tasks of the intervention program. Wright and his colleagues have constructed a large bank of teaching tasks for teachers to use in the program, and selections are made from the bank to ensure that the tasks used are suited to the students' identified needs. Wright, Martland and Stafford (2000) provide a detailed description of this program.

### Mathematics Intervention

While not used in a large number of schools, the Mathematics Intervention program is proving to be one of the most promising programs for assisting students based on diagnosis of their problems. Mathematics Intervention was first implemented in 1993, and aims to identify, then assist, children in the first years of school who are 'at risk' of not coping with mathematics. Mathematics Intervention was developed as a collaborative project between the principal and staff of a primary school in the metropolitan area of Melbourne and university researchers (Pearn, 1999).

The program is based on research about children's early arithmetical learning (see for example, Steffe, Cobb, & von Glasersfeld, 1988; Wright, 1991b; 1994). The initial assessment for Mathematics Intervention program requires teachers to assess the extent of the child's mathematical knowledge by observing and interpreting the child's actions as s/he works on a set task. The interview protocols that allow children to talk about their mathematical strategies are the series of instruments forming part of Mathematics Intervention (Hunting, Pearn & Doig, in press). All teachers involved with the Mathematics Intervention program have attended a course in Clinical Approaches to Mathematics Assessment (Gibson, Doig & Hunting, 1993; Hunting & Doig, 1992) to develop and refine their observational and interpretative skills. The developers of Mathematics Intervention believe that this is a requirement for teachers working with students 'at risk' in mathematics.

In Mathematics Intervention emphasis is placed on the verbal interaction between teacher and children, and between children. Children are withdrawn from their classes and work in groups of no more than three, with a clinically-trained teacher, to assist with the development of their mathematical language skills and co-operative strategies. Each session builds on children's understandings as interpreted by the teacher during the previous session. The Clinical Approaches to Mathematics Assessment course ensures that teachers can observe what the child is doing, interpret the child's actions, act on these actions and then reflect on the intervention. Experience with the Mathematics Intervention program has led to the development of several teaching strategies that allow children to experience success with mathematics (Pearn & Merrifield, 1996, 1998).

# POSSIBILITES IN NUMERACY

If the question is What works?, the answer seems to be Many things!

The possibilities for improving numeracy teaching and learning are legion. Every reported program and strategy implemented to improve numeracy teaching and learning reports at least some success, and this is encouraging.

However to make sense of the wealth of information available about improving numeracy achievement, the following sections deal with different facets of the answer to 'How can we improve student numeracy?'. In some instances the focus is on what is believed to make effective numeracy programs, while other facets are directed at what is believed to make numeracy teachers effective.

Finally the Summary brings together and sorts the ideas and information reported in the earlier sections. This summing up provides a platform from which to start constructing the effective numeracy strategy for effective teachers of numeracy.

## Over-arching possibilities

### Programs

In their report on children with learning difficulties Louden et al. (2000) make the point that many people, teachers included, accept that difficulties in numeracy are normal. There are two consequences of this view: the first is that early identification is not seen to be important, and the second is, according to Louden et al. that this encourages a belief that if literacy is well-taught then numeracy will automatically follow. This report also suggests key general strategies that the authors believe would improve the numeracy achievement for all students. These strategies are that:

- more attention be paid to identification of children who encounter learning difficulties in numeracy;
- there be a commitment to early intervention programs;
- there be a commitment to quality numeracy teaching in the early years;
- there be support for children who continue to encounter numeracy difficulties in the later years of schooling; and
- systems, sectors and schools provide high quality professional development for teachers in numeracy.

The Final Report of the Numeracy Task Force (DfEE, 1998) also has guiding principles for effective numeracy planning and teaching. These include:

- teaching all pupils a daily mathematics lesson;
- teaching mathematics to the whole class at the same time;
- providing lessons that concentrate on the development of numeracy skills;
- promoting participation from, and co-operation between students;
- including oral and mental work within each daily mathematics lesson; and
- providing regular mathematical activities and exercises that pupils can do at home.

### Teachers

Effective teachers of numeracy are not all of one kind, but their general characteristics have been identified by research focusing on what teachers do in numeracy classrooms and what they *believe* about mathematics. For example, Lambdin & Lappan's (1997) study expected teachers to employ lessons that:

- were structured;
- had a focus on the mathematics in the lesson;
- had a discussion drawing together strategies, solutions and mathematical connections;
- pushed students to extract the important mathematics in the lesson; and
- connected and generalised what students had learnt.

The research supporting Cognitively Guided Instruction (CGI) argued that teachers' pedagogical decisions should be made on the basis of:

- an understanding of how children learn mathematical content;
- their knowledge of mathematics;
- their knowledge of children's mathematical development; and
- their own beliefs about all three points above.

In their report on effective teachers of numeracy, Askew, et al. (1997) described their most effective teachers of numeracy as being 'connectionist'; that is to say, teachers who:

- had firm beliefs that under-pinned their practice;
- valued students' ideas;
- based teaching on students understandings;
- understood their own mathematics; and
- emphasised connections between the 'parts' of mathematics.

Although change may be an imperative, research suggests that for most teachers change is stressful, and thus well-planned professional development programs and professional support are essential ingredients of any change-orientated program.

## Possibilities for specific groups of students

### Indigenous students

In *What Works?,* an examination of strategies for improving Indigenous education, McRae et al. (2000) list features, many of which also apply to non-Indigenous students. Factors listed as the most important for effective numeracy learning are that Indigenous students should:

- have fluency in the language of instruction;
- be able to understand the nuances of Standard Australian English;
- have a sense of security, safety and challenge;
- attend school regularly; and
- be engaged in the educational task.

Not all factors are based on what students should do and recommendations for teachers (and systems) include that there is:

- cultural inclusivity;
- a place for local differences, for example dialects;
- an expectation that students can achieve;
- a well-structured program, with achievable goals; and

- a range of learning media used by teachers and students.

This list of 'what works' for Indigenous students is supported by the Improving Numeracy for Indigenous Secondary Students (INISS) program (see Callingham, 1999) in Tasmania, but INISS would add that:

- teachers must have professional development that gives them an understanding of mathematics and children's mathematics; and
- assessment tasks must be directly related to classroom tasks.

Callingham (1999) notes that the INISS program has been effective in improving numeracy for all students, giving weight to the argument that 'what works' for Indigenous students should be available for all.

### Students with learning difficulties

The comments made about learning conditions for Indigenous students is reflected in the report *Mapping the Territory*, Louden et al. (2000) which investigated the state of literacy and numeracy for students with learning difficulties. These students, while needing similar conditions as Indigenous students, also require additional, targetted support. Louden et al. (2000) recommend that this additional support should include:

- early identification and frequent, continuing diagnostic assessment;
- modelling and scaffolded support;
- frequent one-to-one teaching for new learning in a quiet setting;
- breaking the lesson into manageable parts;
- provision of reinforcement;
- positive feedback at time of action; and
- relevant practical experiences.

### The 'at risk' student

Students considered 'at risk' are to some extent included in the previous category, although their difficulties are usually considered not as acute. They are in danger of not progressing as well as they might because their learning is less effective. Many programs exist for these students, and for those in the early years of schooling these programs focus almost exclusively on number. The focus of programs further up the school usually broadens to include other aspects of mathematics, such as space and measurement.

The early years programs that are widely used in Australia are nearly all based on the research of Steffe and Wright, and have as their basis the notion of 'counting stages'. In these programs students are assessed to establish their position within the counting stage framework and a follow-up teaching program planned (see Wright, Martland & Stafford: 2000, for a detailed example).

In essence, programs for 'at risk' students rest on:

- research into the acquisition of counting skills;
- assessment based on the counting stages;
- individualised attention during assessment and instruction;
- a solid period of focused, weekly instruction; and
- a program of instruction over an extended period.

### The early years

Another group for whom special strategies are suggested for their numeracy program to be effective is students in the early years of schooling. Recent research in this area suggests that:

- lessons need to have a conceptual focus;
- greater use be made of open-ended questions;
- students be given more time to explore concepts;
- there be more chance for children to share their strategies;
- as a consequence of higher expectations there should be greater challenge for students;
- there be greater emphasis on making explicit connections at the end of a lesson;
- there be more emphasis on links and connections between mathematical ideas and between classroom mathematics and ‘real life mathematics’; and
- there be less emphasis on formal recording and algorithms.

The overlap in the recommendations for all these groups suggests that investigating the effectiveness of these group-specific recommendations for a broader range of students could be fruitful.

## Summary

The summary of the recommendations derived from research and practice in effective numeracy are in three tables. Each table lists the responsibilities of the student of numeracy, the teacher of numeracy, and the numeracy policy-maker. The responsibilities are those applicable to all students, and the recommendations that apply substantially to only a special group of students are not included here.

***It is the student's responsibility to …***

- *be fluent in Standard Australian English and understand its nuances;*
- *engage with the educational task;*
- *attend school regularly;*
- *use all classroom resources (including the teacher) to a maximum.*

***It is the teacher's responsibility to …***

- *ensure cultural inclusivity;*
- *allow for local differences, for example dialects;*
- *make students feel secure, safe and challenged;*
- *have an expectation that students can achieve;*
- *have a well-structured program, with achievable goals;*
- *use a range of learning media;*
- *use assessment tasks that are directly related to classroom tasks;*
- *identify numeracy problems early;*
- *use frequent diagnostic assessment;*
- *have a solid period of focused instruction;*
- *have lessons with a conceptual focus;*
- *make greater use of open-ended questions;*
- *give students more time to explore concepts;*
- *give students more opportunity to share their strategies;*
- *have more emphasis on links and connections between mathematical ideas and classroom mathematics.*

***It is the system's responsibility to …***

- *ensure that teachers have Professional Development that gives them an understanding of mathematics and children's mathematics;*
- *ensure cultural inclusivity;*
- *establish an expectation that students can achieve;*
- *encourage assessment tasks that are directly related to classroom tasks;*
- *identify numeracy problems early;*
- *use diagnostic assessment;*
- *conduct research into the acquisition of numeracy skills;*
- *use assessment based on the results of numeracy research;*
- *ensure less emphasis on formal recording and algorithms.*

The responsibilities of teachers and systems, derived from effective numeracy research and strategies, can be encapsulated by the following quotation from Malin (1998) who suggests that the teacher who will be most successful is one who expects their children to achieve and:

> who understands their students well, and who see themselves as learners, also open to new understandings from both their students and the parents of their students.
>
> (p.351)

## Future possibilities

While the evidence suggests that levels of numeracy learning in Australia are among the best in the world, there are still areas of concern. Studies, both local and international, have shown that for some groups of Australian students, and for some topics in mathematics, levels of performance could be better. Levels of performance attained by Indigenous students, for example, are consistently lower than that of their non-Indigenous classmates. Summaries of strategies to remedy this have been given earlier but, as Askew (2000) points out for England, much of the evidence is self-referential and therefore the reported outcomes are open to question.

In the United States the same problem exists for well-to-do schools in Illinois. Students at schools in the North Shore area of Illinois achieved very highly in the TIMSS tests although schools recognise that there were still some areas of concern. In an effort to pinpoint the problems and to suggest solution strategies the North Shore schools have compared background factors of their schools (for example amount of homework, number of topics taught per year level) with those of the higher achieving Asian schools (for example, Singapore and Japan). The schools believe that by using the international TIMSS data in this way they will improve students' learning and alter teachers' practices to be more effective (Colvin, 2000).

Many of the reviewed numeracy strategies, and the supporting research, focus on improving teachers' understanding of both mathematics and children's mathematics. It is clear that the importance of teachers' research-based understanding of particular aspects of numeracy and its learning, cannot be under-estimated.

It is with these ideas in mind that the following suggestions are made as starting points for discussing what works and how do we know:

- undertake objective evaluation of strategies currently believed effective;
- conduct research into those strategies that are feasible but as yet untested;
- conduct fundamental research into student learning of specific topics;
- identify school, class and background factors that pinpoint the 'at risk' student through meta-analyses of TIMSS data;
- develop better numeracy assessment for students at all levels;
- investigate extending the Australian Studies of School Performance (ASSP) to identify longitudinal trends;
- conduct a longitudinal study of the strengths and weaknesses of Indigenous students; and
- disseminate to parents and teachers what can be *shown* to work.

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# The ACER
# Core-Funded Research Program

The Australian Council for Educational Research conducts a core program of research funded by an annual grant from the States and Territories and the Commonwealth.

This annual grant allows research to be undertaken into issues of general importance in Australian education and complements research projects commissioned from time to time by individual States, Territories and the Commonwealth.

Priorities for the ACER core research program are reviewed every three years. The three-year program under which this work was completed focused on an over-arching question: What can be done to improve learning outcomes? and addressed five priority areas:

- assessment and reporting to improve learning
- improving literacy and numeracy learning
- improving outcomes for Indigenous students
- teaching practices to improve learning
- vocational outcomes and lifelong learning

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