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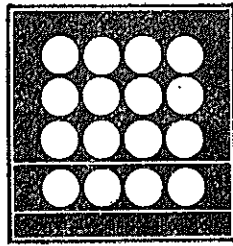
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commonwealth
senior
secondary
scholarships
examination

afternoon session:
thursday
19 july
1973

time allowed:
two hours



comprehension & interpretation (mathematics)

instructions to candidates

This is a test of your ability to use basic mathematical principles and ideas. If a question involves terms or principles which you have not met before, these will be explained in sufficient detail to enable you to answer the question. You may also want to make use of the symbols and formulae printed on this page.

You will obtain the best possible score if you observe the following points:

- (1) Work carefully through the questions in the order in which they are given.
- (2) Don't waste too much time on any one question; if necessary, go on to the next question and come back to the difficult ones later.
- (3) If you think you know an answer, mark it—even if you are not certain you are correct. Marks will **not** be deducted for wrong answers.
- (4) Make sure that you mark the letter you have chosen in the correct line on your answer sheet.

answering

Each question has four alternative answers, represented by the letters **A B C D**. You must choose one answer from these alternatives. Having done so, make sure you mark your answer correctly.

If you want to change an answer, erase your first mark completely. Try to avoid having to make erasures by not answering hastily. Take care that your pencil mark does **not cross** into another row or column, and that there are no marks or smudges on your answer sheet.

Now look through this booklet, but **don't start writing** until the supervisor tells you to do so.

symbols

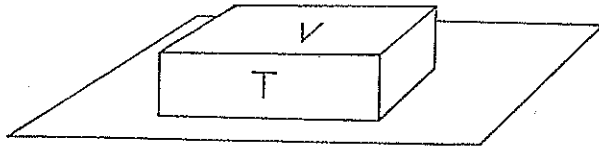
- = means 'is equal to'
- \neq means 'is not equal to'
- > means 'is greater than'
- < means 'is less than'
- \perp indicates that the angle between the two lines is a right angle

formulae

- area of a circle
= $\pi \times$ square of radius, i.e. $A = \pi r^2$
- area of a triangle
= $\frac{1}{2} \times$ base \times height, i.e. $A = \frac{1}{2}bh$

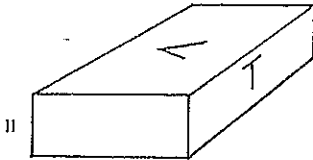
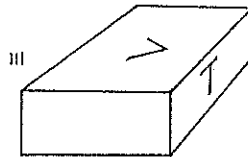
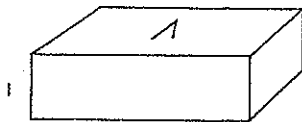
Questions 1 and 2 refer to the following information :

A matchbox has the letter V carefully printed on its top face and the letter T carefully printed on one side face, as shown in the following diagram. No other faces have letters printed on them.



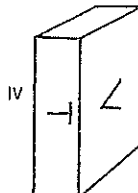
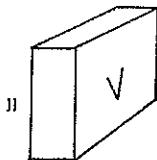
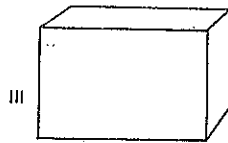
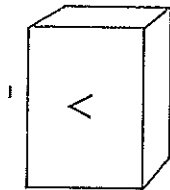
Initially the matchbox is lying on a board with the face labelled V uppermost as shown.

1 If the matchbox is rotated keeping the face V uppermost, which of the following diagrams could be of this particular matchbox?



- A I only
- B II and III only
- C I and II only
- D I and III only

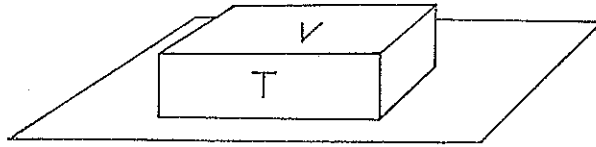
2 If the matchbox is placed in its original position, then rolled forward onto the face labelled T, and then rotated keeping the face T down, which of the following diagrams can not be of this particular matchbox?



- A III only
- B I and IV only
- C II and III only
- D IV only

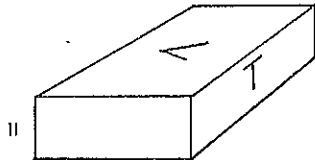
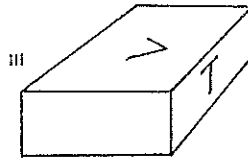
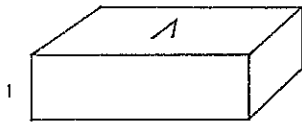
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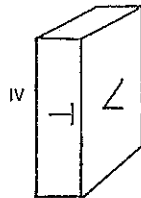
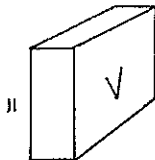
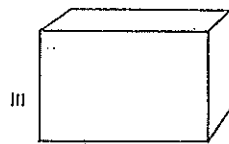
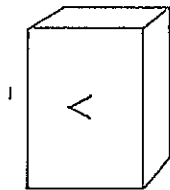
Initially the matchbox is lying on a board with the face labelled V uppermost as shown.

- 1 If the matchbox is rotated keeping the face V uppermost, which of the following diagrams could be of this particular matchbox?



- A I only
 B II and III only
 C I and II only
 D I and III only

- 2 If the matchbox is placed in its original position, then rolled forward onto the face labelled T, and then rotated keeping the face T down, which of the following diagrams can not be of this particular matchbox?



- A III only
 B I and IV only
 C II and III only
 D IV only

Questions 3–5 refer to the following information :

Positive integers m and n are selected two at a time and satisfy the following conditions :

- (i) m is less than n which is less than 7 ;
- (ii) m and n have no common factor other than 1.

3 The number of different fractions $\frac{m}{n}$ which can be formed satisfying these conditions is

- A 36
- B 30
- C 22
- D 11

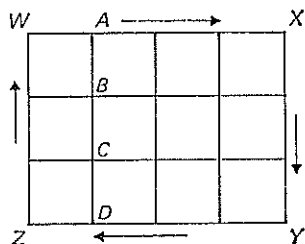
4 In the set of fractions $\frac{m}{n}$ which can be formed satisfying these conditions, the number which appears most frequently as a denominator is

- A 2
- B 3
- C 5
- D 6

5 If p is a member of the set of fractions $\frac{m}{n}$ satisfying these conditions, then so is $1 - p$.

- A This statement is always true.
- B This statement is always false.
- C This statement is sometimes true and sometimes false, depending on the numerical value of p .
- D It is not possible to decide anything about the truth or falsity of this statement.

Questions 6 and 7 refer to the following information :



$WXYZ$ represents a rectangular grid marked out in squares.

6 Audrey sets out from point W and walks once around the perimeter of the grid in the direction shown by the arrows. Graeme sets out from point A at the same time and walks to and fro directly between points A and D at the same speed as Audrey. Where will Graeme be when Audrey returns to point W ?

- A at point A
- B at point B
- C at point C
- D at point D

7 Audrey sets out from point W and walks once around the perimeter of the grid in the direction shown by the arrows. Graeme sets out from one of the points A , B , C , or D at the same time and walks to and fro directly between points A and D at the same speed as Audrey. If Audrey and Graeme meet at point D , from what point did Graeme start walking?

- A point A
- B point B
- C point C
- D point D

Questions 8–11 refer to the following information :

The symbol $[a, b]$ will be used to denote a number pair, where a and b are any real numbers. Two number pairs $[a, b]$ and $[c, d]$ are equal if, and only if, $a = c$ and $b = d$. Number pairs can be ‘multiplied’ as follows :

$$[a, b] * [c, d] = [ac, b + d]$$

where ac denotes a multiplied by c and $b + d$ denotes b plus d . We will use the symbol $*$ to indicate ‘multiplication’ of number pairs.

- 8 $[2, 3] * [-1, 5]$ is equal to
 A $[1, 8]$ C $[-2, 8]$
 B $[-2, 15]$ D $[1, 15]$
- 9 $[a, b]^5$ is equal to
 A $[a^5, b^5]$ C $[a^5, 5b]$
 B $[5a, 5b]$ D $[5a, b + 5]$
- 10 If $[2, 3] * [x, y] = [x, y] * [2, 3]$ then
 A $x = 2$ and $y = 3$ C $2y = 3x$
 B $x = 1$ and $y = 0$ D x and y can take any values.
- 11 If $[a, b]^4 = [1, 1]$ then $[a, b]$ is equal to
 A $[1, 1]$ or $[-1, -1]$ C $\left[1, \frac{1}{\sqrt{2}}\right]$ or $\left[-1, -\frac{1}{\sqrt{2}}\right]$
 B $[1, 1]$ or $[-1, 1]$ D $\left[1, \frac{1}{4}\right]$ or $\left[-1, \frac{1}{4}\right]$

Questions 12–18 refer to tables of numbers called ‘difference tables’.

Here is an example of a difference table :

2		3		10		19		58		117
	1		7		9		39		59	
		6		2		30		20		
etc.										

The numbers in the second row are called the ‘first order differences’ and are obtained from numbers in the first row as follows :

$$3 - 2 = 1, 10 - 3 = 7, \text{ etc.}$$

(Notice that the number to the left is always subtracted from the number to the right.) The third row of numbers (i.e. the second order differences) is obtained from the second row in the same way.

- 12 If this procedure is continued, the set of fourth order differences will be
 A 24 and 18 C -70
 B 32 and -38 D 16

- 13 If an error was made in writing down the numbers in the first row, but it is known that the second order differences should be

6 2 20 40

which one of the entries in the first row must be incorrect?

- A 10 C 58
B 19 D 117
- 14 If the numbers in the first row of a difference table are given by the formula
- $n = 2x^2 + 1,$
- where x takes the values 0, 1, 2, 3, and 4,
which of the following statements is (are) true?
I The first order differences are 1, 3, 9, 19, 33.
II The second order differences are all the same.
III All order differences after the second are zero.
- A I and II only C III only
B I and III only D II and III only

- 15 The following notation for difference tables is sometimes used :

a_1		a_2		a_3		a_4		a_5		a_6
	Δ_1		Δ_2		Δ_3		Δ_4		Δ_5	
		Δ_1^2		Δ_2^2		Δ_3^2		Δ_4^2		
				etc.						

where $\Delta_1 = a_2 - a_1$, $\Delta_2 = a_3 - a_2$, etc. represent first order differences,
where $\Delta_1^2 = \Delta_2 - \Delta_1$, $\Delta_2^2 = \Delta_3 - \Delta_2$, etc. represent second order differences, and so on.
If this notation is used, then Δ_1^4 will be

A a first order difference. C a fourth order difference.
B a third order difference. D always zero.

- 16 If the notation of question 15 is applied to the difference table used in question 12, then $\Delta_2^2 + \Delta_3^2$ will be equal to

- A 130 C 16
B 32 D none of the above.

- 17 It is also possible to use another notation for difference tables as indicated below :

a_1		a_2		a_3		a_4		a_5		a_6
	∇_2		∇_3		∇_4		∇_5		∇_6	
		∇_2^2		∇_3^2		∇_4^2		∇_5^2		∇_6^2
				etc.						

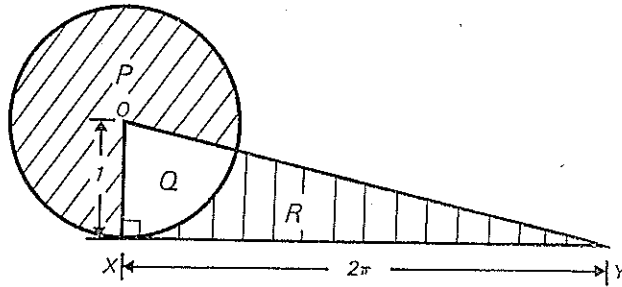
where $\nabla_2 = a_2 - a_1$, $\nabla_3^2 = \nabla_4 - \nabla_3$, etc.
Using this notation, ∇_5^2 will be equal to

- A $a_5 - a_3$ C $a_5 - 2a_4 - a_3$
B $a_5 - 2a_4 + a_3$ D $2a_4 - a_3 - a_5$

- 18 Referring to the two notations outlined in questions 15 and 17, which of the following statements is true?

- A $\nabla_4^2 = \Delta_4^2$ C $\nabla_4^2 = \Delta_2^2$
B $\nabla_2^2 = \Delta_2^2$ D None of the above statements is true.

- 19 In the diagram below, XY , of length 2π units, is a tangent at X to the circle whose centre is O and whose radius is 1 unit. P , Q , and R are areas indicated by the differently hatched parts of the diagram.



Which one of the following statements is correct?

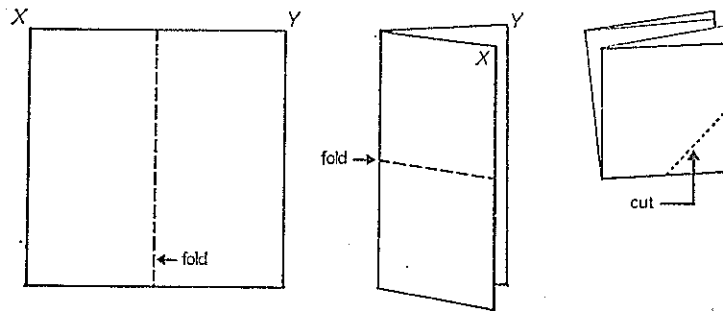
A $Q = \frac{1}{4}(P + Q)$

C $P = R$

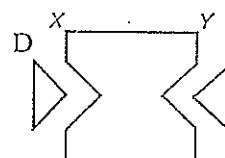
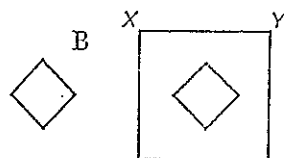
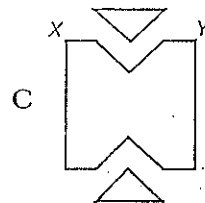
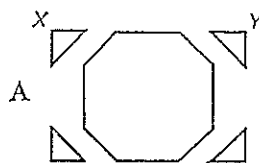
B $Q = \frac{1}{4}R$

D $R = \frac{1}{4}(P + Q)$

- 20 A square of paper is folded twice, as shown in the diagram, and then cut along the dotted line.



After cutting, the paper is opened out. Which of the following shows the resulting shapes?



Questions 21 and 22 refer to the following information :

$R(n)$ denotes a rule by which a number is obtained from n .

The rule is such that

- (1) $R(m + n) = R(m) + R(n)$, where m and n are positive numbers
(2) $R(2) = 1$

21. Which one of the following statements is false?

A $R(1) = 0$

B $R(3) = 1.5$

C $R(4) = 2$

D $R(0.5) = 0.25$

22. Which one of the following statements is true for all values of n ?

A $R(2^n) = n$

B $R(n^2) = 2R(n)$

C $R(2n) = 2R(n)$

D $R(2n) = R(n) + 1$

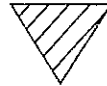
23. If a solid wooden cone like the one depicted below is cut straight through with a saw, which one of the following shapes representing the sawn faces could not be obtained?



A



C



B

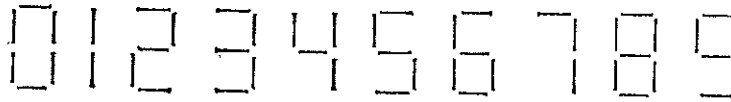


D



Questions 24-26 refer to the following information :

A certain computer has seven elements, some or all of which light up. They light up in combinations to give the patterns shown below, for the numerals 0-9.



The computer 'reads' the patterns in the order shown by the arrows in the diagram below. If an element is off (unlit) it is read as 0 and if an element is on (lit up) it is read as 1 ; so, for example, the number 4 would be read 0011101.



24 How would the number 2 be read?

- A 1010101
- B 1101011

- C 1111100
- D 1101101

25 If the computer's reading sequence breaks down and not all the elements are read (and those which are read may be read in an order which is different from the usual order), it may still be possible to decide from the incomplete display which numeral was intended. Which one of the following reading patterns would allow this?



26 If, as a result of a fault in the machine, two numerals appear together, the effect is that two 'on' signals cancel and the element is observed to be 'off'. Otherwise on + off = on, off + off = off.

If 2 and 4 are superimposed, the resultant pattern is



Questions 27–29 refer to the following information about the ‘numbers’ i , j , and k :

$$i^2 = j^2 = k^2 = -1 \text{ (} i^2 \text{ means } i \text{ multiplied by } i\text{)}$$

$$ij = -ji = k$$

$$jk = -kj = i$$

$$ki = -ik = j$$

27 ijk is equal to

A j

B k

C 1

D -1

28 $jik - kij$ is equal to

A 0

B 1

C 2

D -1

29 In which line of the following argument does the first false statement occur?

A $i^2 j^2 = (ij)^2$

B therefore $i^2 j^2 = k^2$

C but $i^2 = -1$, $j^2 = -1$, and $k^2 = -1$

D so $(-1)(-1) = -1$, that is $1 = -1$

Questions 30–33 refer to the following information:

A collection of blocks has three attributes—colour, size, and shape. The colour may be red, blue, or yellow; the size may be large or small; and the shape may be square, circular, or triangular. Each possible combination of the attributes occurs with equal frequency in the collection, and the collection contains six red triangular blocks.

30 How many blocks are in the complete collection?

A 18

B 54

C 81

D 108

31 What fraction of the total number of blocks is large and circular?

A $\frac{1}{6}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{1}{2}$

32 What fraction of the total number of blocks is neither large nor blue?

A $\frac{1}{6}$

B $\frac{1}{4}$

C $\frac{1}{3}$

D $\frac{2}{3}$

33 How many different kinds of triangular block are there in the collection?

A 3

B 5

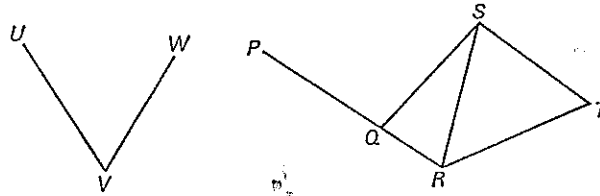
C 6

D 18

Questions 34–37 refer to the following information :

A network is a system of power stations (represented by points P, Q, R, \dots) and supply lines (represented by straight line segments). The supply lines in a network join some (or all) of the power stations.

Two stations are said to be connected if it is possible to transmit power from one to the other via supply lines. In the following network, P and S are connected, and P and W are not.

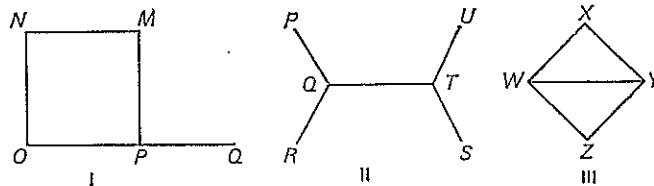


- 34 A system of power stations has K connected to L , but L not connected to M . Which one of the following deductions is valid?
- A K is connected to M .
 B K is not connected to M .
 C K may or may not be connected to M .
 D The system does not form a network.
- 35 A system of power stations has F not connected to G , and G not connected to H . Which one of the following deductions is valid?
- A F is connected to H .
 B F is not connected to H .
 C F may or may not be connected to H .
 D The system does not form a network.

Questions 36 and 37 refer to the following additional information :

If the removal of a power line results in the two stations which it joined being no longer connected, the power line is called a **vital link**. A **tree network** is a network in which every station is connected to every other station and the network has at least one vital link.

36 Which of the following is (are) tree networks?



- A I only
 B II only
 C I and II only
 D I, II, and III

- 37 A tree network connects n power stations. What is the least number of supply lines that there could be in the network?
- A $n - 1$
 B n
 C $2n$
 D There is insufficient information to decide.

Questions 38–43 refer to the following information :

David Dimble plays with sets of fractions which are made up as follows :

The Dimble set of order n (which is written D_n) consists of all positive fractions less than 1, such that the numerator and denominator have no common factor and the denominator is less than or equal to n .

For example, $D_3 = \{\frac{1}{2}, \frac{1}{3}, \frac{2}{3}\}$

The number of fractions in D_n is denoted by N_n . So $N_3 = 3$.

The sum of the fractions in D_n is denoted by S_n . So $S_3 = 1\frac{1}{2}$.

David was able to show that, for all n , $S_n = \frac{1}{2}N_n$.

38 N_4 is equal to

A 4

B 5

C 6

D 7

39 N_2 is equal to

A 0

B 1

C 2

D 3

40 If $m > n$, then

A N_m must be greater than N_n .

B N_m cannot be less than, but may be equal to N_n .

C N_m can be less than, but not equal to N_n .

D A, B, and C are all false.

41 $S_7 - S_6$ is equal to

A $1\frac{1}{2}$

B 2

C $2\frac{1}{2}$

D 3

42 $\frac{S_m + S_n}{N_m + N_n}$

A is equal to 1

B is equal to $\frac{2}{m+n}$

C is equal to $\frac{1}{2}$

D cannot be determined from the information given.

43 If $m \neq n$, $\frac{S_m - S_n}{N_m - N_n}$

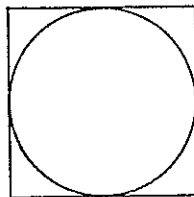
A is equal to 1

B is equal to $\frac{2}{m-n}$

C is equal to $\frac{1}{2}$

D cannot be determined from the information given.

Questions 44 and 45 refer to the following diagram which shows a circle and its circumscribing square :



- 44 As the radius of the circle increases and the circumscribing square expands accordingly, the ratio $\frac{\text{area contained by circle}}{\text{area contained by square}}$
- A remains constant.
 B increases.
 C decreases.
 D increases until the circle's radius is 1 unit, and then decreases.
- 45 As the radius of the circle increases and the circumscribing square expands accordingly, the ratio $\frac{\text{perimeter of circle}}{\text{perimeter of square}}$
- A remains constant.
 B increases.
 C decreases.
 D increases until the circle's radius is 1 unit, and then decreases.

Questions 46–50 refer to the following information :

In the Annual Snake Gully Snail Race there are ten contestants, numbered 1–10. It is stated (by an experienced judge of snails) that their respective probabilities of winning are as shown below :

snail	1	2	3	4	5	6	7	8	9	10
probability of winning	0.48	0.20	0.10	0.08	0.04	0.04	0.03	0.01	0.01	0.01

- 46 The probability that the winner's number is a multiple of 3 is
- A 0.05
 B 0.10
 C 0.15
 D 0.30
- 47 The probability that the winner's number is a multiple of 3 or a multiple of 2 (or both) is
- A 0.30
 B 0.45
 C 0.49
 D 0.70
- 48 The odds against the occurrence of an event is the ratio of the probability of the non-occurrence of the event to the probability of occurrence of the event. The odds against snail 2 winning are
- A 4 : 1
 B 5 : 1
 C 9 : 1
 D 10 : 1
- 49 If snail 3 is withdrawn from the race by order of the stewards, then the probability that snail 2 wins
- A is increased.
 B stays the same.
 C is decreased.
 D cannot be determined.
- 50 Given that the place getters were snails 2, 5, and 8, in some unknown order, the probability that the winner was snail 5 is
- A 0.04
 B 0.16
 C 0.25
 D 0.33

Questions 51–53 refer to the following information :

If n is an integer, we define its d -cube, $n^{(3)}$, as follows :

$$n^{(3)} = n(n-1)(n-2)$$

51 Which of the following statements is/are true for all integers n ?

- (i) $n^{(3)}$ is an even number
- (ii) $n^{(3)}$ is exactly divisible by 3
- (iii) $n^{(3)}$ is exactly divisible by 6

- A (i) only
- B (ii) only

- C (i) and (ii) only
- D (i), (ii), and (iii)

52 $89^{(3)}$ is equal to

- A 68 124
- B 681 376

- C 681 384
- D 6 814 746

53 If $n^{(3)} = 10\,626$, then

- A $n = 18$
- B $n = 23$

- C $n = 26$
- D $n = 33$

Questions 54–57 refer to the following information :

$S_n = 1 + 2 + 3 + \dots + n$, that is, S_n is the sum of the first n counting numbers, and

$T_n = 1 + 3 + 5 + \dots + 2n - 1$, that is, T_n is the sum of the first n odd numbers.

For example, $S_4 = 1 + 2 + 3 + 4 = 10$, and

$$T_3 = 1 + 3 + 5 = 9$$

54 T_n is equal to one of the following for all values of n .

Which one?

- A $2S_n + n^2$
- B $S_n + n - 1$

- C $2S_n - n$
- D $2S_n - n(n-1)$

55 S_{2n} is equal to one of the following for all values of n .

Which one?

- A $2S_n + n^2$
- B $S_n + 2n$

- C $2S_n$
- D $2S_n - n(n-1)$

56 T_{2n} is equal to one of the following for all values of n .

Which one?

- A $2S_{2n} + n^2$
- B $2S_{2n} - n$

- C $2S_{2n} + 4n^2$
- D $2S_{2n} - 2n$

57 T_n is equal to one of the following for all values of n .

Which one?

- A n^2
- B $n(n-1)$

- C $\frac{1}{2}n(n+1)$
- D $n(n+1)$

