Why teach mathematics?

Paul Ernest considers some fundamental questions: why learn mathematics, why teach mathematics, and what do we want students to get out of it?

The questions of why to teach mathematics and why to learn mathematics raise a complex set of issues including: who is doing the asking; what are their aims, goals, interests and ideas of relevance; and who are the target students being discussed? Questions of aims always have a social context.

Some of the reasons we teach mathematics are the basic functional ones that aim at developing student competence. Below I list seven capabilities that I see as desirable for learners to develop. Some are widely accepted, some are more controversial, but all are underpinned by a consideration of what it is we want learners to gain from their school mathematics learning experiences.

NECESSARY MATHEMATICS

The three categories below constitute dimensions of useful or necessary mathematics for all or some, primarily for the benefit of employment and society from an economic perspective, as well as sustaining mathematics and mathematical interests themselves.

Functional numeracy This involves being able to deploy mathematical and numeracy skills adequate for successful general employment and functioning in society. This is a basic and minimal requirement for all at the end of schooling, excluding only those few with some preventive disability.

Practical, work-related knowledge This is the capability to solve practical problems with mathematics, especially industry and work-centred problems. This is not necessary for all, for the depth and type of problems vary across employment types, and most occupations requiring specialist mathematics also provide specialist training. However, a strong case can be made for schools to provide the basic understanding and capabilities upon which further specialist knowledge and skills can be built.

Advanced specialist knowledge This involves understanding specialist advanced knowledge of mathematics, in high school or university. This is not a necessary goal for all adults, but is desired or needed by a minority of students. Clearly this option must be available, and indeed more students should be encouraged to pursue it, but it should not dominate or distort the school mathematics curriculum for all.

SOCIAL AND PERSONAL MATHEMATICS

In comparison to the capabilities of 'necessary' mathematics, there are mathematical capabilities that are less directly utilitarian. Instead they are more to do with personal, cultural and social relevance, although ultimately I believe they have powerful incidental benefits for society.

Mathematical problem posing and solving This involves deploying mathematical knowledge and powers in both posing and solving mathematical problems, and is the area of greatest potential for creativity in school mathematics. While problem solving is widely endorsed, although too often focussing more on routine than non-routine problems, problem posing has been more neglected. This is the articulation and formulation of questions and problems to be solved. It can enable us to see mathematical connections between superficially diverse questions and topics, and to frame questions by analogy. This is inevitably linked to the next capability.

Mathematical confidence This includes being confident in one's personal knowledge of mathematics, feeling able to use and apply it, and being confident in the acquisition of new knowledge and skills when needed. The most directly personal outcome of learning mathematics, it uniquely involves the development of the whole person in a rounded way encompassing both intellect and feelings. Effective knowledge and capabilities rest on freedom from negative attitudes to mathematics, and on the feelings of enablement, empowerment as well as enjoyment in learning and using mathematics. These latter lead to persistence in solving difficult mathematical problems, as well as willingness to accept difficult and challenging tasks. In my view, the domain of attitudes, beliefs and values is one of the most important psychological dimensions of learning mathematics and we need to pay much more attention to it in school. Seemingly insignificant incidents can switch a learner on or off mathematics, and we need to be more sensitive to this in our teaching.

Social empowerment through mathematics The knowledge of mathematics learned in school should enable learners to function as numerate, critical citizens who are able to use their knowledge in social and political realms, for the betterment of both themselves and society as a whole. Social empowerment through mathematics involves critically understanding the uses of mathematics in society: to identify, interpret, evaluate and critique the mathematics embedded in social, commercial and political systems and claims, in advertisements – such as in the financial sector – and in government and interest-group pronouncements. Every citizen needs to understand the limits of such uses of mathematics, and where necessary reject spurious or misleading claims. Ultimately, such a capability is a vital bulwark in protecting democracy and the values of a humanistic and civilised society.

Critical mathematical literacy or critical citizenship through mathematics is a major topic on its own and the critical mathematics education movement has sprung up to deal with theory and practice in this area. To learn more about social empowerment through maths, see Danish mathematician Ole Skovsmose's *Towards a Philosophy of Critical Mathematics Education* or my chapter 'Why teach Mathematics?' in the 2000 compendium *Why Learn Maths?* I could expand on this topic to fill up the end of this article, but there is another capability that is more neglected.

THE APPRECIATION OF MATHEMATICS AS AN ELEMENT OF CULTURE

The last of my proposed seven aims or capabilities is the development of mathematical appreciation, as opposed to the inculcation of skills and capabilities. The latter dominate much of the study of mathematics in school and beyond. Mathematical capability and the appreciation of mathematics can be likened to the study of language and literature. Mathematical capability is like being able to use language effectively for oral and written communication, whereas mathematical appreciation resembles the study of literature, in that it concerns the significance of mathematics as an element of culture and history, with its own stories and cultural pinnacles, and the artefacts of mathematics are understood in that context, just as great texts are in literature.

Appreciation of mathematics This comprises the appreciation of mathematics itself, and its role in history, culture and society in general. It involves a number of dimensions and roles, which can include the following:

- Appreciating the role of mathematics in life and work, the importance of mathematics in commerce, economics (such as the stock market), telecommunications, information and communication technology, and the role it plays in representing, coding and displaying information. Also how mathematics is forever becoming more central to, but also more deeply and invisibly embedded in, all aspects of our daily life and experience.
- Having a sense of mathematics as a central element of culture, art and life, present and past, which permeates and underpins science, technology and all aspects of human culture. This extends from symmetry in appreciating elements of art and religious symbolism, to understanding how modern physics and cosmology depend on algebraic equations such as Einstein's $E=mc^2$.

Being aware of the historical development of mathematics, the social contexts of the origins of mathematical concepts, its symbolism, theories and problems is important in itself, as well as enhancing the study of mathematics. The development of mathematics is inseparable from the most important developments in history, from ancient societies in Mesopotamia, Egypt and Greece (number and tax and accounting, geometry and surveying) via medieval Europe and the Middle East (algorithms and commerce, trigonometry and navigation, mechanics and ballistics) to the modern era (statistics and agriculture-biology-medicine-insurance, logic and digital computing, media and telecommunications). Virtually all of these areas are underpinned by what is arguably the most important family of concepts invented by humankind (especially in India), namely number and place value. Students first meet these concepts in their early years, and keep deepening their grasp of them throughout their years of study.

Mathematics is a central theme in the development of civilizations over the past five millennia. There is also a very interesting branch of study called ethnomathematics that looks at informal culturally embedded mathematical concepts and skills from cultures around the globe, both rural and urban. In addition to throwing light on the cultural origins of mathematics, this also is a great source of methods and examples to enrich the teaching and learning of mathematics.

Having a sense of mathematics as a unique discipline, with its central branches and concepts as well as their interconnections, interdependencies, and the overall unity of mathematics is another key element of appreciation. This includes its central roles in many other disciplines including, most notably, biology, chemistry, physics, computing and information and communication technology. After many years spent studying mathematics, learners should have some conception of mathematics as a discipline, including understanding that there is much more to mathematics than number and what is taught in school.

Understanding the ways that mathematical knowledge is established and validated through proof, as well as the limitations of proof is also important. I believe this should include introduction to the philosophy of mathematics: understanding that there are big questions and controversies about whether mathematics is discovered or invented, about the certainty of mathematical knowledge and about what type of things mathematical objects are. I discuss these issues in my 1990 book, *The Philosophy of Mathematics Education*. Being aware of controversies over proof and the foundations of mathematics supports a more critical attitude to the social uses of mathematics.

Learners should gain a qualitative or intuitive understanding of some of the big ideas of mathematics such as pattern, symmetry, structure, proof, paradox, recursion, randomness, chaos and infinity. Mathematics contains many of the deepest, most powerful and exciting ideas created by humankind. These extend our power to think and imagine, as well as providing the scientific equivalent of poetry, offering noble, aesthetic, and even spiritual experiences.

Are these aims concerning appreciation feasible for school? One must be careful about prejudging what is appropriate or accessible to school children. One of the arguments against the appreciation of big ideas like infinity is that it is too difficult for school children. But many an interested eight-year old will happily discuss the infinite size of space, or the never-ending sequence of the natural numbers. Let's not underestimate what learners are capable of, and indeed what we as mathematics education professionals are capable of. After all, there are around 2,000 hours of school mathematics to play with, over the course of compulsory schooling. Surely we can afford to be a little more ambitious in the variety of ideas and approaches to mathematics we offer!

LINKS:
For the Philosophy of Mathematics Education Journal*, visit http://people.exeter.ac.uk/PErnest*
For more information on the ACER research conference, visit www.acer.edu.au/research_conferences

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Paul Ernest will speak at the Australian Council for Educational Research (ACER) 2010 research conference to be held in Melbourne, 15-17 August. The conference theme is 'Teaching Mathematics? Make it count: What research tells us about effective mathematics teaching and learning. It will consider approaches to teaching that develop the mathematical proficiency of students and that catch and hold their interest in mathematics from the early years through to post-compulsory education.