Effective teaching: Lessons from mathematics

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Abstract

Teachers of mathematics face a double challenge. The first challenge is addressing the short-term needs of learners in meeting expected standards. But there is also the challenge of the long-term needs of learners developing productive dispositions towards the unanticipated mathematics that they will encounter beyond schooling. Teaching that concentrates only upon delivering a pre-determined body of mathematical knowledge may meet the short-term needs, but not the longer ones. Teaching that attends to the processes of learning and doing mathematics is more likely to meet both sets of needs. The Australian curriculum for mathematics encapsulates these process aspects through the four proficiencies of fluency, reasoning, problem-solving and understanding. This presentation examines the research behind learning these proficiencies and the implications for teaching practices. I will look at teaching practices that appear to be effective in helping learners develop these proficiencies and also at what may be barriers to such practices being more widely adopted.

Introduction

ACARA (Australian Curriculum and Assessment Reporting Authority) sets out three overarching aims for the mathematics curriculum, one of which being to ensure that students:

> are confident, creative users and communicators of mathematics, able to investigate, represent and interpret situations in their personal and work lives and as active citizens. (ACARA, 2011)

In bringing about this aim, the curriculum has two dimensions: the content strands and the proficiencies. The content strands are familiar: number and algebra, measurement and geometry, statistics and probability. Perhaps less familiar and possibly more challenging to current models of mathematics teaching are the four proficiencies that cut across the content:

- Fluency
- Understanding
- Problem solving
- Reasoning.

These proficiencies describe ‘how content is explored or developed, that is, the thinking and doing of mathematics’ (ACARA, ibid.) and ‘the actions in which students can engage when learning and using the content’ (ACARA, ibid.). Given the unpredictability of the mathematics that students of today may need in their lives of tomorrow, these proficiencies are important in promoting the ‘mathematical habits of mind’ (Cuoco, Goldenberg and Mark, 1996) and productive dispositions (National Research Council, 2001) that learners will need to engage with when meeting new mathematics.

Viewing the proficiencies as the actions through which students learn the content presents a challenge to the popularly held view that they need to learn the content first – addition, equivalent fractions, algebraic manipulations or whatever – and only subsequently apply it to solving problems, or to be able reason about it. It also presents a challenge to teaching.

I have some difficulty with understanding as an ‘action’ – I can develop understanding, I can draw on understanding, I can demonstrate understanding, but I’m not clear how I ‘do’ understanding. I prefer to think of understanding as the outcome of doing the other proficiencies – engaging in problem solving, reasoning about the ‘why’ of mathematics and being fluent in the ‘how’ of mathematics are the building blocks of understanding. In what follows I will therefore focus on fluency, problem solving and reasoning.

The call to think about proficiencies as ‘actions’ can sound contradictory to the everyday use of ‘proficient’ as a degree of expertise. We would not describe someone stumbling through ‘chopsticks’ as a proficient piano player. But learning to play the piano involves engaging in actions before one is fully skilled in them – there is no waiting to become fully fluent in, say, playing scales before being expected (and encouraged) to play a tune. Becoming a proficient piano player means working with all of the musical proficiencies – scales, reading music, playing sonatas – from the beginning. Becoming a proficient mathematician requires working with all of the mathematical proficiencies – fluency, problem solving, reasoning and understanding – from the beginning. And by mathematician here I mean anyone using mathematics in his or her life. Everyone is a mathematician.

Taking the proficiencies seriously means moving from seeing school mathematics as a body of knowledge for learners to acquire to seeing it as an activity for learners to engage in – in the words of Brent Davis, moving from seeing mathematics as *pre*formed to mathematics as *per*formed (Davis & Sumara, 2006).

Teaching through mathematical proficiencies

Teaching mathematics through engaging learners in the actions of the proficiencies has pedagogic implications. In particular, no one-size-fits-all pedagogy enables the enactment of all proficiencies. Effective teaching arises out of repertoires of pedagogies. Two particularly salient aspects of such repertoires are varying the organisation of groups and the orchestration of classroom dialogue.

Teaching and group work

Generally group work is promoted as good for learning, but nuanced research findings indicate the importance of grouping students in particular ways for particular purposes. Classroom grouping decisions need to take into account:

- group size
- group interactions
- group composition
- group culture
- and how each of these interact with intended learning outcomes and the learning tasks set.

Group size

In an extensive review of research, Kutnick and colleagues summarise the evidence for the relationship between group size and learning task (Kutnick, Sebba et al., 2005). They identified paired work as best for developing understanding, provided the partners trust each other and can work well together. Trust and cooperation seem to be more important to considerations when selecting pairs to work together than factors such as matching on attainment levels (more on this below). Small groups appear to be best suited to enrichment tasks.

Practice and revision, however, appears best done individually as tasks can be differentiated and time on task is more focused on the necessary practising. Thus, aspects of mathematics teaching focused on developing fluency are best matched to individual work (and perhaps set as homework, since practice should not require a teacher to hand).

Group interactions

A key feature of the effective group work is the development of what emerges from the task being more than the sum of the individual efforts. Researchers have variously referred to this as *groupsense* (Ryder & Campbell, 1989), or *intersubjectivity* (Rogoff, 1990; Wertsch, 1991). In Mercer's terms, group members move beyond interacting, to *interthinking* (Mercer, 2000).

Group composition

Studies of learning outcomes reveal that a predictor of who may learn most from group work is the participant asking the most questions of the others in the group. The evidence also shows that the person answering the most questions makes the next highest learning gains(Webb, 1989).
Webb's research shows that group composition in terms of range of attainment can affect the extent of and participation in such questioning and answering. Groups studied where the range of attainment was narrow were characterised by scant questioning and answering going on. Where group members are similar in attainment it seems that either they get on with tasks on the assumption that everyone in the group knows what to do, or they assume that others in the group will not be able to help. If the attainment range was broad, the participants at the extremes of the range engaged in most of the questioning and answering, thus limiting the opportunity for those in the middle to gain as much from the group interactions. Thus, it seems that groups need to have some range of attainment, but not too broad a range.

Group tasks

Tasks for pairs or groups to work on need to be carefully chosen and beyond the grasp of any individual member of the group, linking back to Davis and Sumara's (2006) point about planning for the collective: if tasks are chosen on the basis of being appropriate for the level of individual attainment, they may not be sufficiently challenging to provoke interthinking.

This was exemplified by a project with a school in the East End of London. Standards (as judged by National Test results) were extremely low and the teaching largely focused on trying to raise the attainment of individuals. Working with the school over two years, we focused on paired work and providing challenging tasks for pairs (once we got over the resistance from the learners who were unused to this style of teaching), which students could not have succeeded in individually. Although not the only intervention in the school, standards rose dramatically and students typically began to comment on how easy they found the National Tests, which indeed were much simpler than tasks worked on in class.

To summarise, tasks need to be chosen that require 'resources (information, knowledge, heuristic problem solving strategies, materials and skills) that no single individual possesses, so that no single individual is likely to solve the problem or accomplish the task objectives without at least some input from others' (Cohen, 1994).

Group culture

For groups to function well, research also indicates that all group members must believe that both their own and their partners' contributions are important. Meyers(1997) found that 'individuals exert less effort in groups when they believe that their work is not critical to the collective'.

We cannot take this mutual valuing of contributions for granted as research by Jenny Young-Leveridge from the University of Waikato New Zealand shows. Students she interviewed expressed the importance of sharing their solution methods with their peers as well as the contradictory view that listening to others' explanations was not that important!

Despite the evidence that good group work leads to results that are more than the sum of individual efforts, the evidence is that while students may sit together in groups, the enactment of effective group work is still limited. Why might this be so?

One possible reason is the dominance of discourse of teaching being about meeting individual needs. Davis and Sumara (2006) argue that teaching needs to attend to the needs of the group and that with that in place, the needs of individual learners then fall into place. If we shift attention to planning for the group rather than the individuals in the group, then the research into group learning outcomes indicates a shift is required in thinking about the level of difficulty of tasks selected. It seems commonsense to assume that mixed attainment groups or pairs working together may lead to the lower attaining students advancing towards the level of attainment of the higher attaining students, but those higher attaining students not gaining as much from the experience. Research does show, however, that even when group members have differing levels of attainment, the more advanced students can progress as much as their less advanced peers (Damon & Phelps, 1988, Schwarz, Neuman, & Biezuner, 2000) – the old saying of 'two heads being better than one' appears to hold true. Conversely, closely matched groups have been found to make little progress.

In many mathematics lessons a range of solutions may be presented but as a form of show-and-tell rather than to provoke dialogue. Ideas need to 'bounce off' each other for mathematics to emerge (Davis & Sumara, 2006), which will not happen if students are not attending to, building on or arguing against each others' explanations. Good group work and appropriate tasks can provoke socio-cognitive conflict – differences amongst group members – with research findings supporting the impact of this on the learning of individuals (e.g. see Ames & Murray, 1982; Bearison, Magzamen & Filardo, 1986). All this points to the importance of classroom dialogue in effective teaching.

Dialogue and effective teaching

Much of the advice in the mathematics educational literature is similar to the notion of 'accountable talk' that Lauren Resnick and colleagues introduced to highlight that classroom talk must be judged against something. Classroom talk can be accountable to three things: building the community, reasoning and knowledge (Michaels, O'Connor & Resnick 2008).

Resnick's research shows that developing accountable talk directed to building community is possibly the easiest to implement in classrooms. Teacher moves like asking 'Who agrees with what Lynne has just said?' 'Jennie, you had a different idea, how does that fit?' 'Who can re-explain in their own words what Russell has just said?' can change the dynamic of classroom dialogue from one of 'show-and-tell' to one of collective engagement with the mathematics.

The talk that then arises also has to be accountable to reasoning – the arguments and ideas learners produce must be commensurate with the logic of mathematical argument. And the talk must also be accountable to knowledge: the mathematics that emerges must eventually be correct. Resnick suggests, perhaps surprisingly, that it is easier to encourage talk that is accountable to reasoning than it is to produce talk that is accountable to knowledge. She bases this claim on the observation that children can produce well-reasoned arguments but grounded in ideas that are mathematically incorrect. For example, a ten-year-old I once met reasoned cogently that 9 was an even number as nine cubes could be split into three equal groups: the logic of his reasoning was correct, but it didn't fit with the mathematically accepted definition of even.

Finally, despite the evidence showing the power of dialogue in promoting learning, there is also continuing evidence of the dominance of closed questions in mathematics lessons that do not provoke the sort of dialogue that would lead to socio-cognitive conflict and individual learning. A seminal study by Stein and colleagues hints at why this might be so (Stein, Grover & Henningsen, 1996). Working together, the researchers and teachers planned a series of lessons designed to engage learners in cognitively challenging mathematical tasks. When the researchers watched these lessons actually being enacted in classrooms they found that only one-third of the lessons actually maintained the challenge as they played. In two-thirds of the lessons the challenges were reduced to following procedures that the teachers pointed out to the learners or in some cases the lessons became non-mathematical. One of the factors in lessons that maintained the challenge was teachers 'sustained pressure for explanation and meaning'. Sustained pressure – effective teaching doesn't come easy.

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